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Recovery of the spatial state of the ionosphere using regular definitions of the TEC at the network of continuously operating GNSS stations of Ukraine

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Introduction

Application of a network of continuously operating reference stations to determine numerical characteristics of the Earth's ionosphere allows creating an effective technology to monitor the ionosphere regionally. This technology is intended to solve both scientific problems concerning the space weather, and practical tasks such as providing coordinates of the geodetic level accuracy.

Formulation of the problem

For continuously operating reference GNSS stations, the results of the determined ionization identifier *TEC* (Total Electron Content) that describes the number of ions in the atmosphere on the line between the ground station and the moving satellite accumulate. On the one hand, this data reflects the state of the ionosphere during the observation; on the other hand, it is a substantial tool for accuracy improvement and reliable determination of coordinates of the observation place.

Formulation of the problem

Thus, it was decided to solve a problem of restoring the spatial position of the ionospheric state or its ionization field according to the regular definitions of the ***TEC*** identifier, i.e. ***STEC*** (Slant *TEC*).

We propose one of the possible solutions that is based on the application of the regularized approximation of functions with numerous variables.

Formulation of the problem

Initial data to restore the ionization field:

coordinates of reference stations

$$X_i^{st}(t_k), Y_i^{st}(t_k), Z_i^{st}(t_k), i=\overline{1,n_k}, k=\overline{1,K} \quad (1a)$$

coordinates of GNSS satellites

$$X_j^{sp}(t_k), Y_j^{sp}(t_k), Z_j^{sp}(t_k), j=\overline{1,m_k}, k=\overline{1,K} \quad (1b)$$

STEC values between the station *i* and satellite *j*

$$s_{ij}(t_k), i=\overline{1,n_k}, j=\overline{1,m_k}, k=\overline{1,K} \quad (1c)$$

Further we used from 19 stations in the Western Ukraine

Formulation of the problem

Where

t_K -time of the *STEC* measurement;

K - number of measurements;

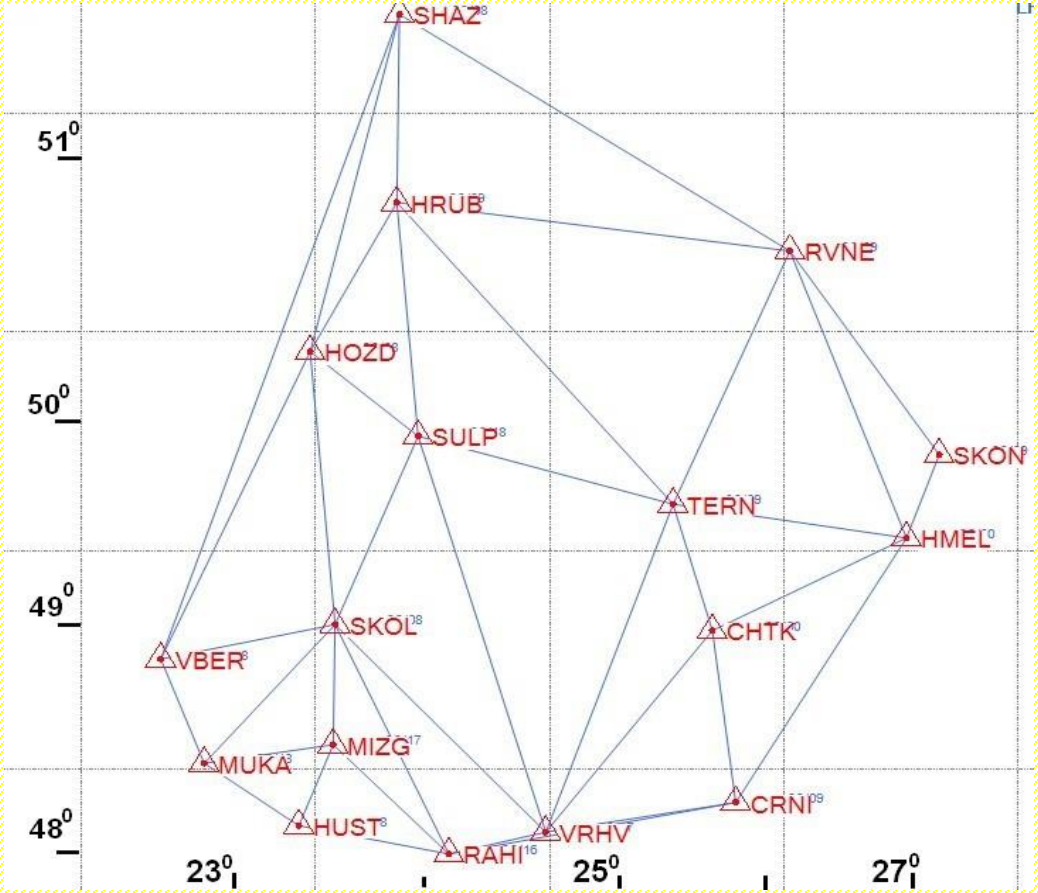
i - station number; m_k

j - satellite number;

n_k, m_k number of stations and satellites during the measurement k, respectively.

Further we used data from 19 stations in the Western Ukraine.

Formulation of the problem



Formulation of the problem

The solution to this problem is to define the ionization field $v = v(x, y, z, t)$

for the area where the stations are located x, y, z

during the time $t \in [t_1, t_K]$

Restrictions and assumptions for use of the GNSS measurements

The coordinates of an individual station (1a) and available satellite *j* (1b) define the line segment that connects the point on the Earth's surface with the satellite. This line segment comes through the Earth's ionosphere as well. **One of the assumptions** in this case is that the ionosphere layer has an effective thickness that is defined by the sub-ionospheric point *H*. According to this assumption, **all ionized atoms are located on the surface of some sphere with the radius defined by the sub-ionospheric point.**

Restrictions and assumptions for use of the GNSS measurements

We divide the part of the specified line segment into *N-1* equal segments, thus, getting *N* equally located nodes that lie on a beam from the station to the satellite below the sub-ionospheric point (fig.1)

$$\bar{x}_{ijl}^{k} = \bar{x}_{ijl}(t_k), \bar{y}_{ijl}^{k} = \bar{y}_{ijl}(t_k), \bar{z}_{ijl}^{k} = \bar{z}_{ijl}(t_k) \quad (2)$$

Restrictions and assumptions for use of the GNSS measurements

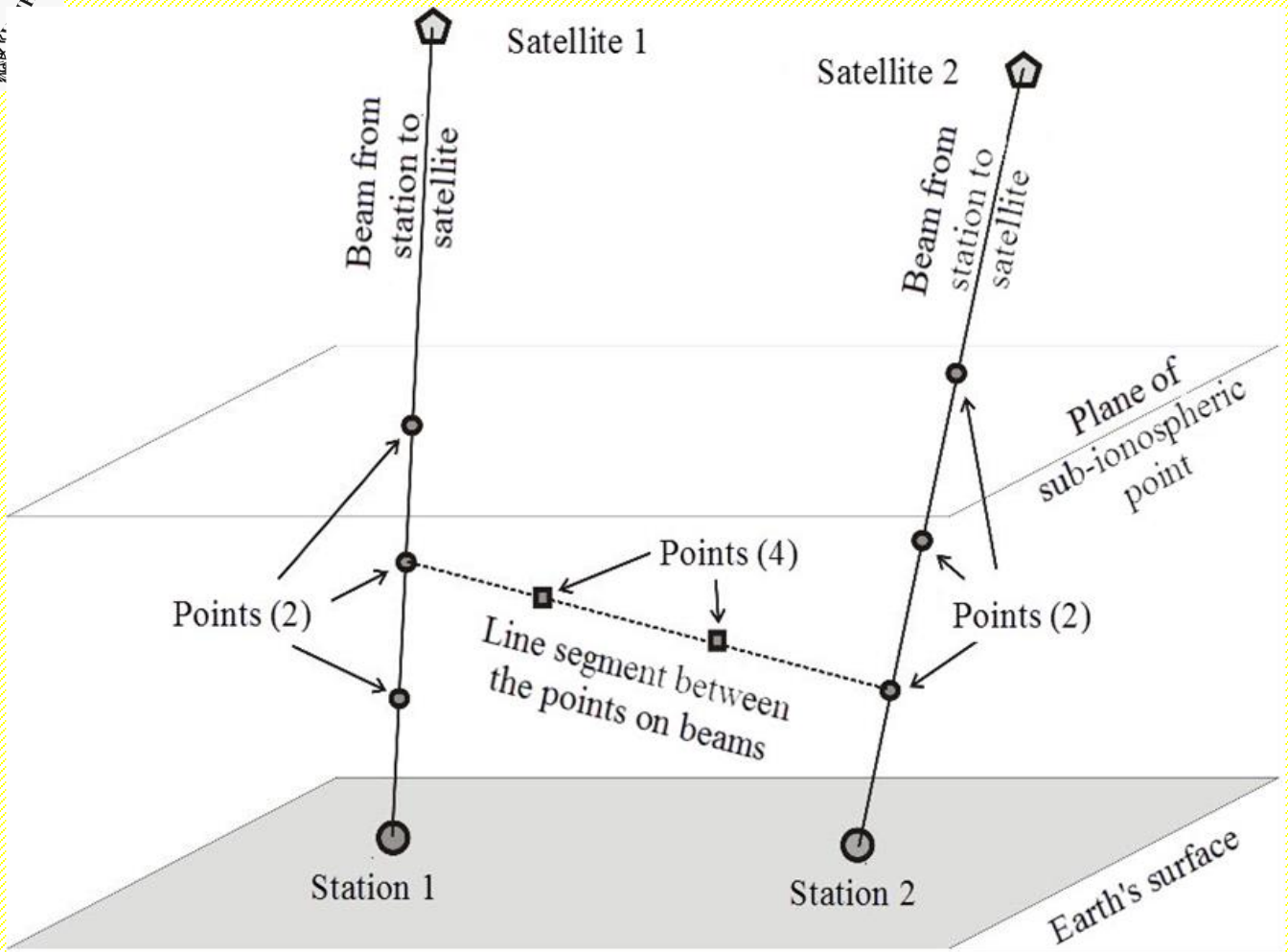


Fig. 1. Spatial location of the nodes

Restrictions and assumptions for use of the GNSS measurements

Where $\overline{x}_{ijl}^{k}, \overline{y}_{ijl}^{k}, \overline{z}_{ijl}^{k}$

are spatial coordinates of a *l*-point on a beam between *i* station

and *j* -satellite, $(i=\overline{1,n_k}, j=\overline{1,m_k}, l=\overline{1,N}, k=\overline{1,K})$

Restrictions and assumptions for use of the GNSS measurements

Supposing that the state of the ionosphere changes evenly along the beam between the station and the satellite, the ionization field indicator in each node (2) can be described as:

$$\bar{v}_{ijl}^{k} = s_{ij}\left(t_k\right)/N \qquad (i=\overline{1,n_k},\ j=\overline{1,m_k},\ l=\overline{1,N},\ k=\overline{1,K}) \quad (3)$$

Where $\bar{v}_{ijl}^{k}$ ionization parameter in point *l* on a beam between station *i* and satellite *j*.

Restrictions and assumptions for use of the GNSS measurements

Expression (3) describes the ionization values determined experimentally. GNSS-observations show that the beams from satellites to stations at one point in time lie mostly in certain directions only, while there are few of them that lie in other directions or none at all (fig.2).

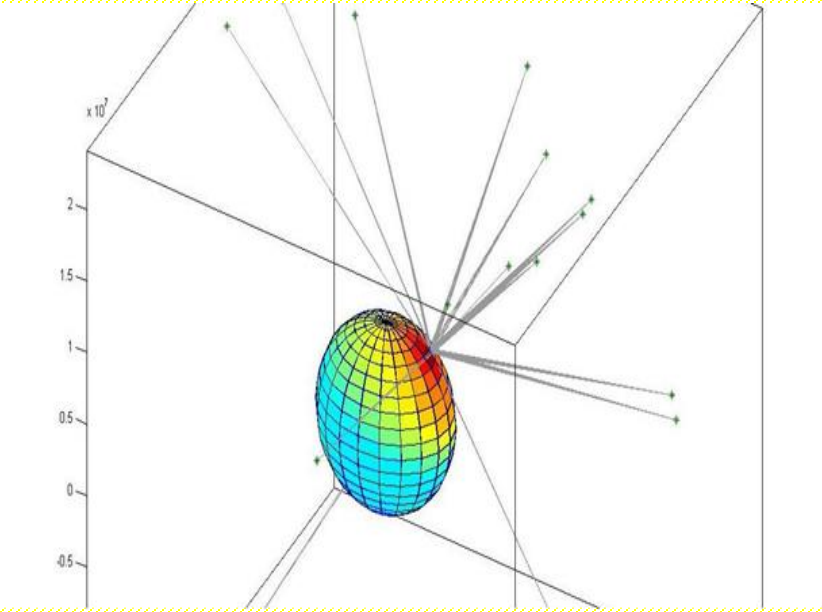


Fig. 2. Common view of beams from the station to the satellites according to (1a and (1b)

Restrictions and assumptions for use of the GNSS measurements

This fact makes the conditions of the interpolation problem worse. For their improvement, we connected all the points (2) on the beams between the stations and satellites and on the formed line segments we defined internal equally located nodes that do not lie on the boundaries of the line segments (see fig.3):

$$\breve{x}_{ijpq}^{kr}, \breve{y}_{ijpq}^{kr}, \breve{z}_{ijpq}^{kr} \quad (4)$$

$\breve{x}_{ijpq}^{kr}, \breve{y}_{ijpq}^{kr}, \breve{z}_{ijpq}^{kr}$ spatial coordinates of the *r*-point on the line segment between the nodes, *M*-number of internal nodes on a line segment between the points on the beam.

Restrictions and assumptions for use of the GNSS measurements

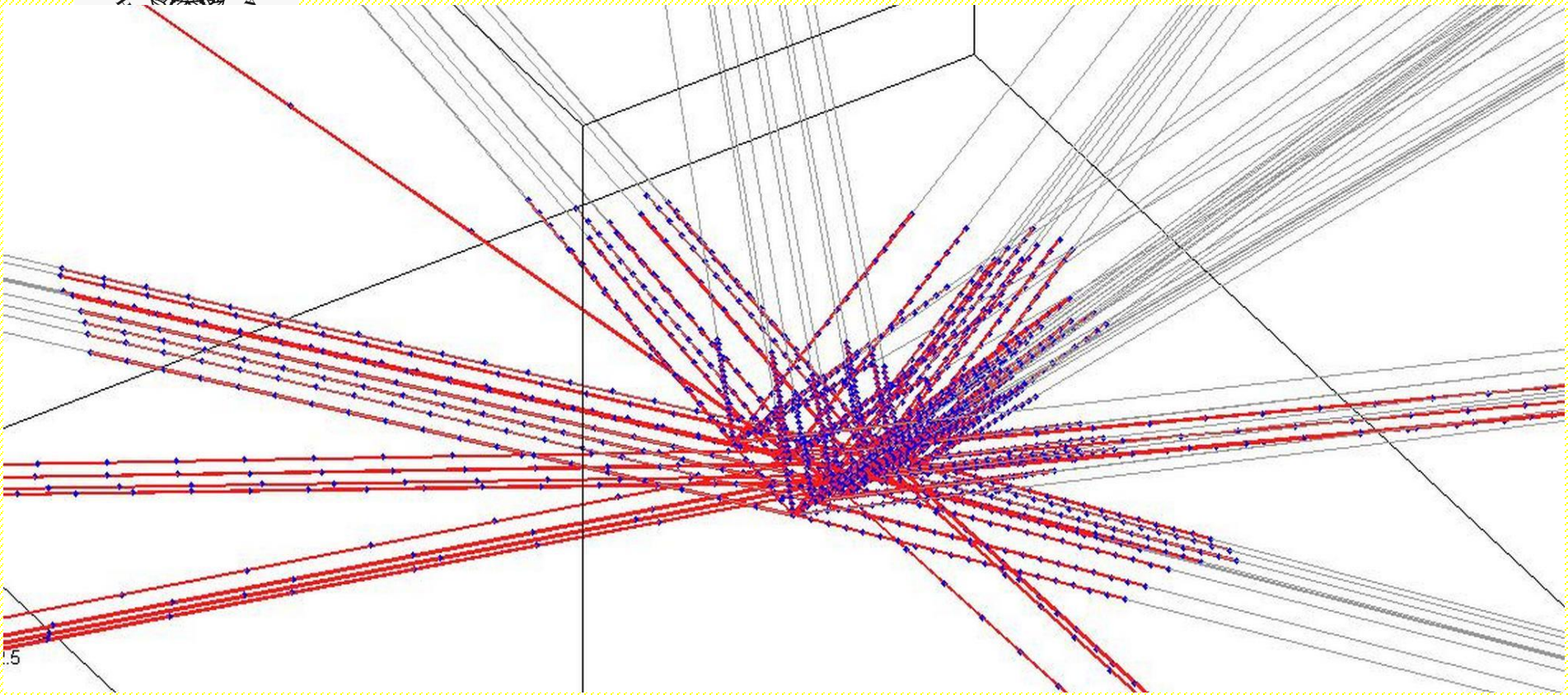


Fig. 3. Beams from stations to satellites (in grey), their sub-ionospheric line segments (in red) and points (2) on them (in blue)

Restrictions and assumptions for use of the GNSS measurements

Ionization parameters in the nodes (4) are defined using linear interpolation value of this indicator on a line segment between the points

$$\breve{v}_{ijpq}^{kr}(i=\overline{1,n_k},\ j=\overline{1,m_k};\ p,q\in[1,N];r=\overline{1,M};k=\overline{1,K}\ (5)$$

Expression (3) describes ionization in the nodes (2) located along the beams between the stations and satellites. Expression (5) describes ionization in the nodes (4) that lie between such different beams. **It was found** that data from expressions (2-5) **is not enough** to restore the ionization field as approximating functions deviate greatly from the observational ionization values beyond the nodes (2), (4).

Method for determining ionization using *STEC*

For practical purposes, **we need to define** ionization in the spatial area:

$$x \in [x_{\min}^k, x_{\max}^k], y \in [y_{\min}^k, y_{\max}^k], z \in [z_{\min}^k, z_{\max}^k]$$

where the area boundaries are defined by the extreme points of the set of nodes (2) that are located on the beams between the stations and satellites:

$$x_{\min}^k = \min_{ijl} \overline{x}_{ijl}^k, y_{\min}^k = \min_{ijl} \overline{y}_{ijl}^k, z_{\min}^k = \min_{ijl} \overline{z}_{ijl}^k$$

$$x_{\max}^k = \max_{ijl} \overline{x}_{ijl}^k, y_{\max}^k = \max_{ijl} \overline{y}_{ijl}^k, z_{\max}^k = \max_{ijl} \overline{z}_{ijl}^k$$

Method for determining ionization using *STEC*

We divide line segments that describe the rectangular area (6) into *L-1* smaller segments and determine the coordinates of the equally located nodes:

$$\widehat{x}_i^k, \widehat{y}_j^k, \widehat{z}_l^k, i, j, l = \overline{1, L} \quad (7)$$

To restore the ionization field in the area (6), a new condition needs to be imposed: **ionization derivatives with respect to the coordinates must be minimal in the points (7).**

Such condition reduces strong deviations of the approximating function beyond the nodes (2), (4). **It should be noted** that the solution to the problem of the ionization field restoration is in finding the ionization values in the nodes (7).

Method for determining ionization using *STEC*

The discrete dependency of the ionization from the values of three spatial coordinates is approximated by the exponential polynomial from numerous arguments:

$$v(x,y,x)=P_k(x,y,x)$$

The structure of the approximating basis was selected in such a way that the argument exponents **are close to 1**. It is empirically known that **this improves** the extrapolation of the simulated values in the nodes (7).

Method for determining ionization using *STEC*

To find approximation coefficient using the data from (8) and (9), we used identification problems regularized by minimizing the stabilizing Tikhonov functional and reduction of the approximating basis. **However**, such approximation has **an acceptable error** of approximation only in the identification nodes (2), (4) and beyond them deviates **greatly from the approximated value in the points** (7).

Method for determining ionization using *STEC*

Thus, to restore the ionization field, **additional measures were taken**. An artificial argument that depends nonlinearly on *x,y,z* was added to the arguments of the polynomial:

$$P_k(x,y,z,r)=\sum_{|i+j+l+p|<R} c_{ijlp} x^{\lambda i} y^{\lambda j} z^{\lambda l} r^{\lambda p} \quad (10)$$

Method for determining ionization using *STEC*

Identification problem intended to determine the polynomial coordinates for a separate calculation has two conditions: **approximation of values** (9) in the points (8) and **minimization of the polynomial derivative** with respect to its argument in the points (7)

$$\min_{c_I^k} \sum_{a=1}^{A} \left[v_a^k - P_k(x_a, y_a, z_a, r_a) \right]^2 + \alpha \sum_I \left(c_I^k \right)^2 \quad (11)$$

$$\min_{c_I^k} \sum_{i,j,l,p=1}^{L} \left(0 - \sum_{q=x,y,z,r} P_k^q(\hat{x}_i, \hat{y}_j, \hat{z}_l, \hat{r}_p) \right)^2 + \alpha \sum_I \left(c_I^k \right)^2 \quad (12)$$

Method for determining ionization using *STEC*

To solve (11), (12), the reduction of the approximating basis. To reduce the deviations of the approximating polynomial from the measured ionization values, the first-degree (R=1) polynomial was chosen and minor deviations $\lambda \in [0.7, 1.3]$ were applied

Method for determining ionization using *STEC*

Multiple solving of the problems (11) and (12) for all measured data lead to such **interim conclusions**:

-if polynomial exponents of numerous arguments are close to 1, then approximation basis found using the reduction of the polynomial exponent while solving the problem (11), (12) for the values of an individual measurement *k* (1) provides an acceptable approximation for all measurements;

-if the exponent of the approximation polynomial differs greatly from 1 the reduction of the approximation polynomial exponent for each measurement leads to obtaining different approximation bases. This does not lead to substantial improvement of the approximation accuracy (10) and mostly makes the accuracy of the expression (12) worse.

Method for determining ionization using *STEC*

From the results of these computational experiments, it can be concluded that to restore the ionization field, it is advisable to use the polynomial (10) with **the exponent *R=1*** and multiplier λ that is **slightly less than 1**. For other conditions, we need **substantial costs** for computational resources to determine the approximation basis and coefficient c_I^k for each of the measurements separately.

Results of the experimental restoration ionization

To experimentally determine the changes in the ionization field in time, we took **k=46** measurements from **272 days** in 2013, ***STEC* values** with time interval of **15 sec**. during the first **12** minutes from the beginning of the day that were determined during the GNSS observations at **17** continuously operating stations of the ZAKPOS network.

Results of the experimental restoration ionization

Exponents of the polynomial arguments determined for the most measurements and the values of the approximation coefficients for a measurement *k*=1

Table 1

№	Exponent x	Exponent y	Exponent z	Exponent r	Coefficient c_I^1
1	0	0	0	0	7.623044
2	0	0	0	0.769231	0.004773
3	0	0	0.769231	0	–0.000469
4	0	0.769231	0	0	–0.003179
5	0.769231	0	0	0	–0.010377
6	0	0	–0.769231	0	–6.092696

Results of the experimental restoration ionization

Using cubic Hermite spline for interpolation (function pchip in Matlab), we obtained the approximation coefficients values for each second. Using the interpolation by the fifth-degree spline, their continuous values $c_i(t), t \in [t_1, t_K]$ were determined.

Figure 4 shows the common dependencies of these approximation parameters from time.

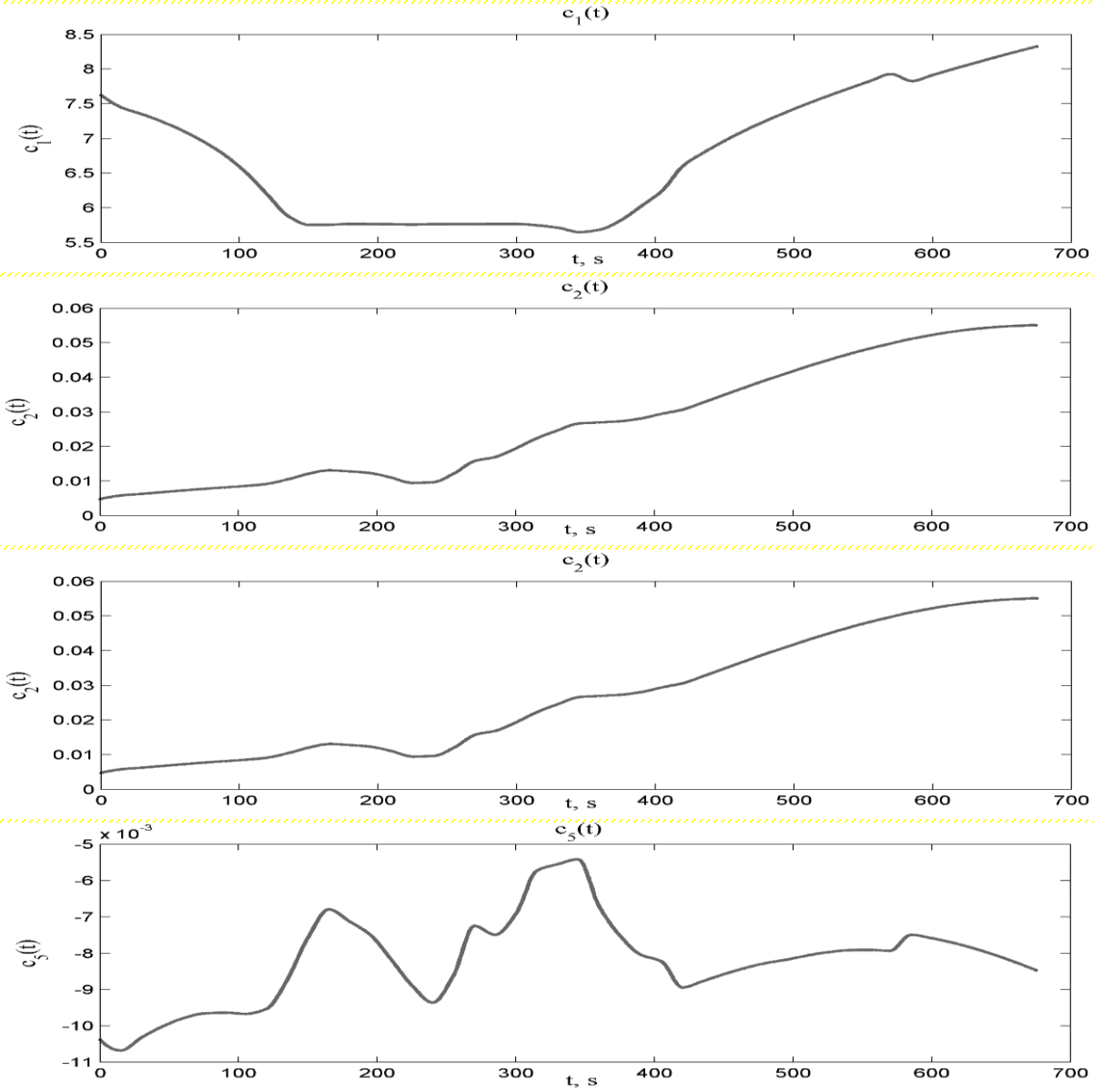


Fig. 4. Time dependencies graph of approximation coefficients $c_1(t)$ (a), $c_2(t)$ (b), $c_3(t)$ (c), $c_5(t)$ (d)

Results of the experimental restoration ionization

Figure 4a-c shows that parameters gradually change over time. This change depends on angle altitude of certain satellites and their ascent and descent, namely changes in number of satellites.

Only one parameter $c_5(t)$ changes relatively very quickly. This indicates that the problem of restoration of $c_i(t)$ using data (1) to determine the reduced (regularized) approximation basis common to all observations is incorrect.

Results of the experimental restoration ionization

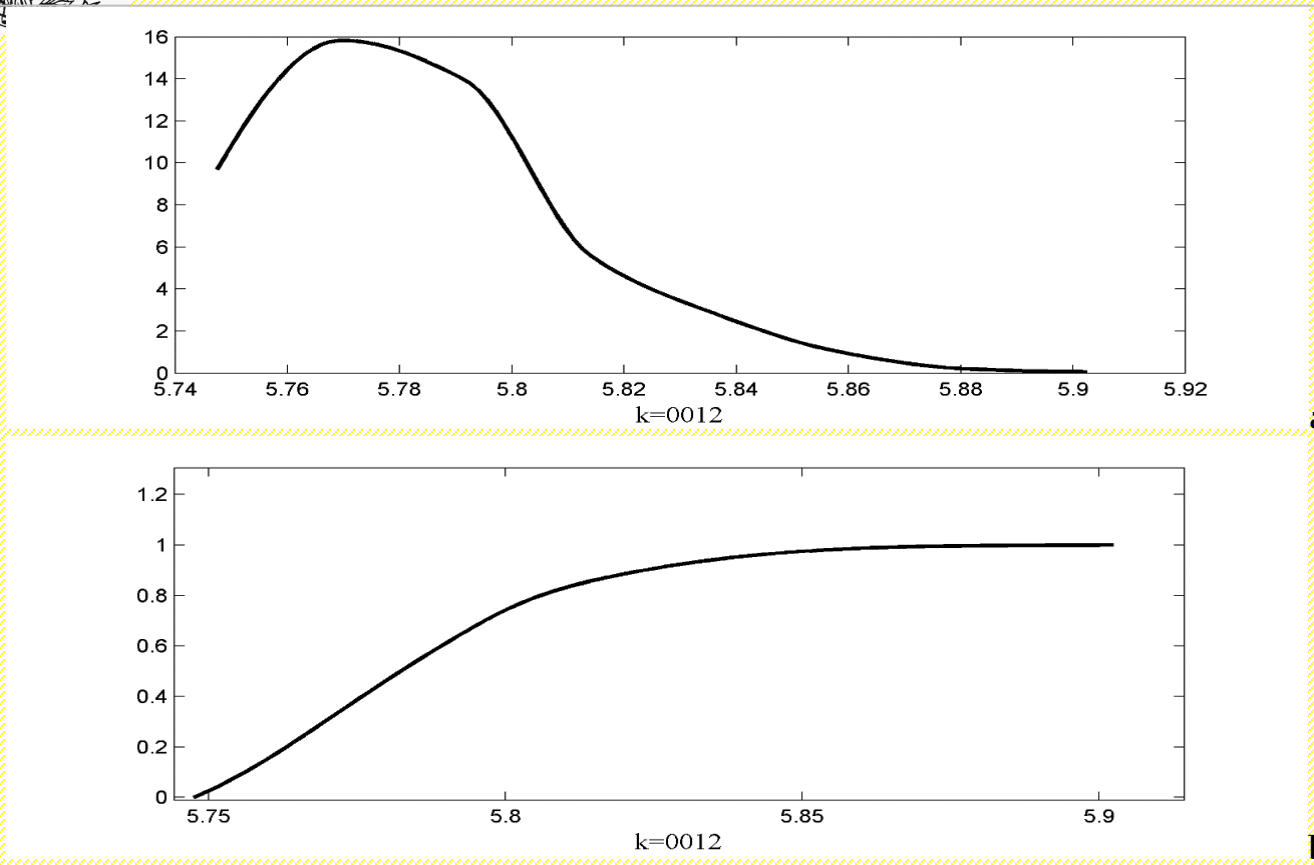
However, the above solution shows that the change speed of $c_5(t)$ is limited.

This fact confirms a good choice in the approximation basis for all observations $k=\overline{1,K}$ (during all time $t \in [t_1, t_K]$)

Results of the experimental restoration ionization

Acceptable solutions to (11) and (12) are distribution (probability density) of the needed ionization in the points (7) that has a central maximum or is close to even or linear distribution. Figure 5 shows common distribution graphs and functions of the ionization distribution $v(x, y, z, t_{12})$ determined using the polynomial (10) in the points (7).

Results of the experimental restoration ionization



a)

b)

Fig. 5. Common view of graphs for probability density (a) and distribution function (b) of the ionization value $v(x, y, z, t_{12})$ restored in the rectangular area (14)

Results of the experimental restoration ionization

To restore the change in the ionization field, we need to determine continuous dependencies of the area center coordinates (7) $x_c(t), y_c(t), z_c(t), (t \in [t_1, t_K])$ from time using approximation by spline. Its shown on figure 6.

Results of the experimental restoration ionization

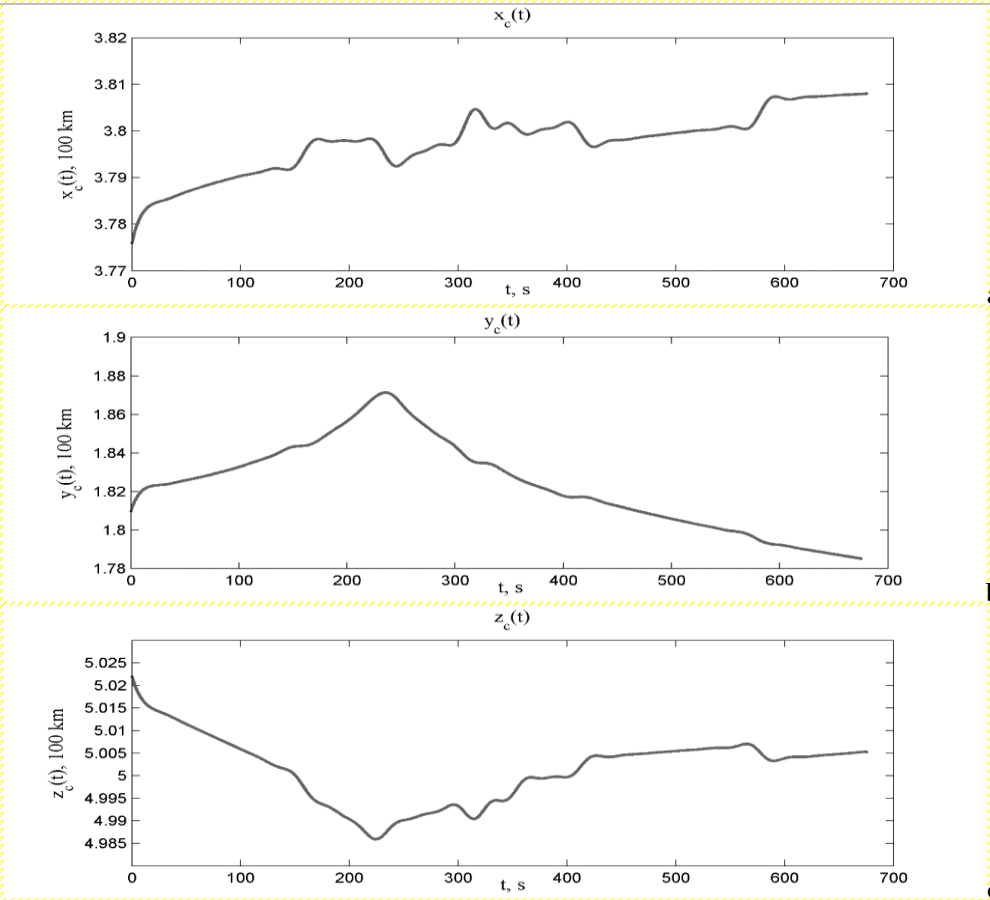


Fig. 6. Time dependency graphs of the center of the ionization restoration area $x_c(t)$ (a), $y_c(t)$ (b), $z_c(t)$ (c)

Results of the experimental restoration ionization

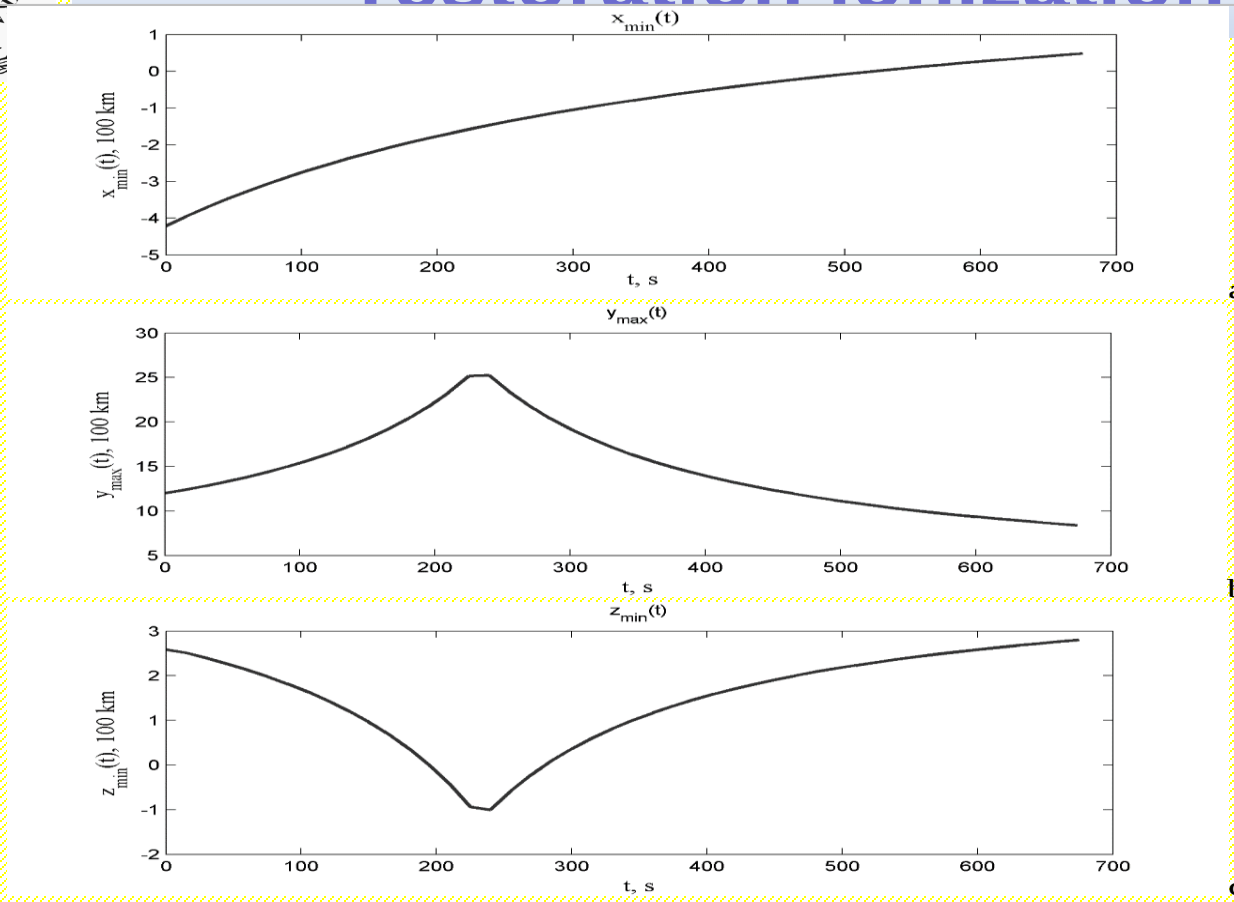


Fig. 7. Common view of time dependency graphs of the changes in boundaries of the ionization restoration area:
$x_{\min}(t)$ - (a), $y_{\max}(t)$ - (b), $z_{\min}(t)$ - (c)

Results of the experimental restoration ionization

Figure 6 shows the shifts of the center of the rectangular area with irregular fluctuations.

This can be explained by the movement of satellites and discrete division of sub-ionospheric line segment of the beam from the station to satellite.

This indicates that the ionization field depends on the algorithm parameters.

Results of the experimental restoration ionization

Extreme values of these boundaries were defined for the ionization field restoration:

$$\bar{x}_{\min} = \max_{k\in[1,K]} x^k_{\min};\ \bar{y}_{\min} = \max_{k\in[1,K]} y^k_{\min};\ \bar{z}_{\min} = \max_{k\in[1,K]} z^k_{\min};\ \bar{x}_{\max} = \min_{k\in[1,K]} x^k_{\max};$$

$$\bar{y}_{\max} = \min_{k\in[1,R]} y^k_{\max};\ z_{\max} = \min_{k\in[1,K]} z^k_{\max}$$

They describe rectangular area

$$x\in[\bar{x}_{\min},\bar{x}_{\max}],\ y\in[\bar{y}_{\min},\bar{y}_{\max}],\ z\in[\bar{z}_{\min},\bar{z}_{\max}], \qquad (13)$$

for which the ionization values during the whole period of observations $t \in [t_1, t_K]$ are restored.

Results of the experimental restoration ionization

We divide the line segments (13) into *L-1* segments and determine the coordinate values for equally located nodes:

$$\hat{x}_i, \hat{y}_j, \hat{z}_l \ (i, j, l = \overline{1, L}). \qquad (14)$$

Ionization values in the nodes (14) are computed using the polynomial (10) with parameters $c_i(t)$ that depend on time: $v(x, y, z, r) = \sum_{ijlp} c_{ijlp}(t) x^{\lambda i} y^{\lambda j} z^{\lambda l} r^{\lambda p}$ (15)

Where *i,j,l,p*-are the indexes of the polynomial coefficients

Results of the experimental restoration ionization

In (15), continuous time functions $c_i(t)$ are defined using splines.

According to (15), we determine the ionization at an arbitrary time $t \in [t_1, t_K]$ in the arbitrary point (13).

It was found that most often, the ionization increases with an altitude and there is a shift of spherical areas with reduced or increased ionization.

Results of the experimental restoration ionization

Sometimes such areas stop shifting and start moving in the opposite direction and mix. Restoring the spatial dynamics of ionization (15) **models complex processes** of electric charge movements in the ionized air.

The described method is based on the interpolation of the coefficients of the polynomial from numerous arguments (10). **It can be used for data in (8) and (9) that leads to the same reduced approximation basis.** This method is described briefly in the end-algorithm

Results of the experimental restoration ionization

This method is described briefly in the end-algorithm

Results of the experimental restoration ionization

Using this algorithm, the results of the ionization field restoration were obtained. In particular, this algorithm was applied with the following parameters: number of points on the beam from the station to satellites N=3; number of points between the points on different beams M=1. With such parameters in the expression (11), the number of approximation nodes exceeds 55 thousand.

Results of the experimental restoration ionization

Number of nodes along the area boundary of the ionization restoration is L=20. Using this value, the number of minimization nodes of the derivative by the polynomial from numerous arguments in (12) is 9261. An increase in the above-mentioned parameters causes severe computation complications in problems (11) and (12) and does not improve the accuracy of its solution.

Results of the experimental restoration ionization

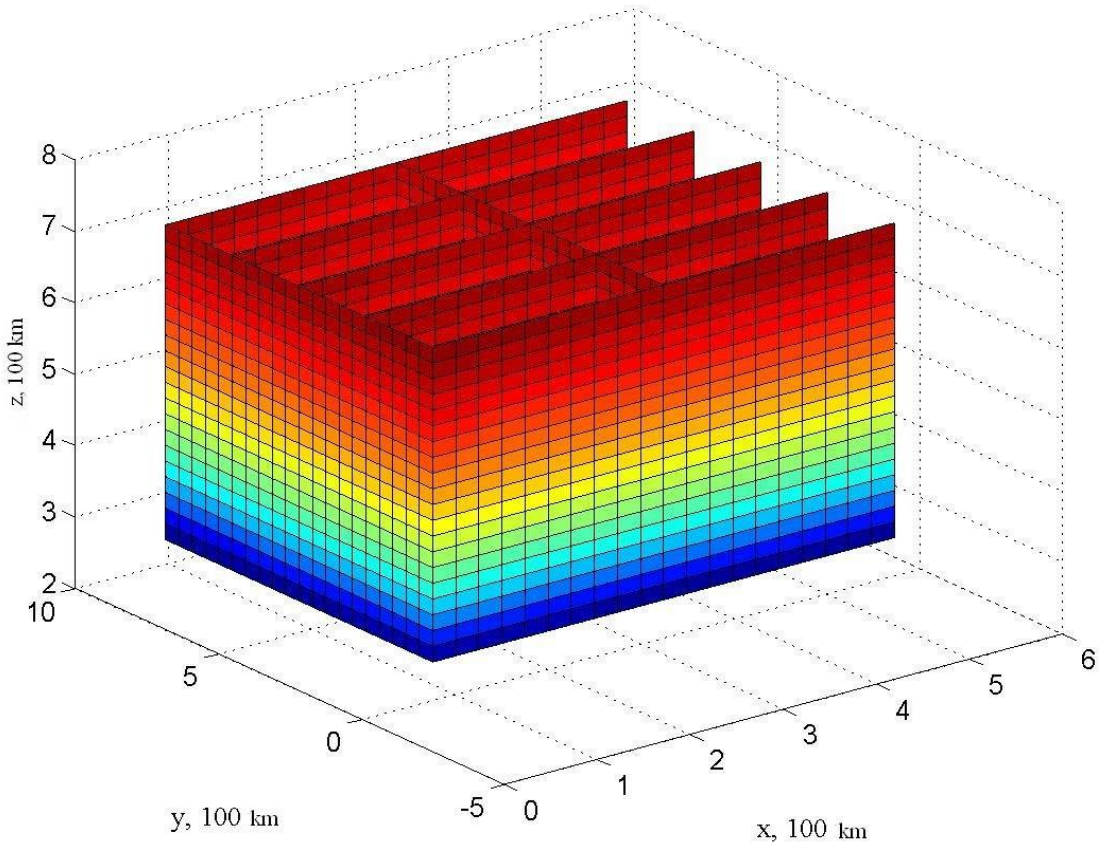


Fig. 8. Graphs of the ionization dependency from the spatial coordinates (in 100km) in the moment $t_{38} = 555$ s

Results of the experimental restoration ionization

Statistical characteristics, that describe 46 results of the solution to the problem (11) and (12) are provided in table 2.

- This table shows such characteristics:
- standard deviation (SD) of the absolute approximation error in 46 results of the problem (11);
- SD of the relative approximation error in 46 results of the problem (11);
- average ionization in the approximation nodes (8) computed for 46 moments in observation from polynomial (10);
- average ionization in the approximation nodes (experimental values from 46 observational moments) (9);
- average relative approximation error by module for 46 solutions to the problem (11)

Results of the experimental restoration ionization

Statistical characteristics for 46 results of the solution to the problem (11) obtained using common approximation basis **Table 2**

Name of identifier	Parameters					
	The smallest value	The bigges value	Average value	Median	Distribution mode	SD
SD of the absolute approximation error in (11)	1.3892	1.4652	1.4198	1.4226	1.3892	0.016779
SD of the relative approximation error in (11)	0.15017	0.15737	0.15247	0.15230	0.15017	0.001354
Average ionization in the approximation nodes (8) computed from polynomial (10)	5.7050	5.8206	5.7377	5.7354	5.7050	0.021957
Average ionization in the approximation nodes (8), experimental values (9)	5.7049	5.8206	5.7377	5.7352	5.7049	0.021971
Average relative approximation error (11) (by module)	0.21556	0.22512	0.21967	0.22016	0.21556	0.002148

Results of the experimental restoration ionization

In particular, table 2 shows that the average values of the experimental and model (obtained from approximation) **values are close for all measurements**. The relative accuracy of the problem (11) solution for an individual measurement is approximately 21% (with dispersion 0.21% for all measurements). The standard deviation of this error is 15% (with dispersion 0.13% for all measurements). This means that using the common approximation basis, we obtained ionization approximation (11) with relatively low accuracy (21%) **but the error of such approximation varies a little for each of the measurements**.

Results of the experimental restoration ionization

This proves **the efficiency of applying the common approximation basis** for regularized approximation of the atmosphere ionization when **using the polynomial from numerous arguments** with coefficients dependent on time. It should be noted that before **we added a new argument to the polynomial** (10), the approximation accuracy was worse.

Other ways to expand or change the approximation basis (described above) do not influence the accuracy parameters for ionization field restoration.

Algorithm recovery of the spatial state of the ionosphere

1. Obtain the coordinates of the stations (1a), satellites (1b) and STEC values (1c).

2. Determine the altitude of the sub-ionospheric point.

3. Select a number of points N on the beams from the stations to satellites located lower than the sub-ionospheric point.

4. Compute (2) the coordinates $\bar{x}_{ijl}^{k}$, $\bar{y}_{ijl}^{k}$, $\bar{z}_{ijl}^{k}$ of the points located on the beams between the stations and satellites lower the sub-ionospheric point ($i=\overline{1,n_k}$; $j=\overline{1,m_k}$; $l=\overline{1,N}$; $k=\overline{1,K}$).

5. Compute (3) the value $\bar{v}_{ijl}^{k}$ in the points (2) (located on the beams between the stations and satellites lower than the sub-ionospheric point) that are defined in step 4 ($i=\overline{1,n_k}$; $j=\overline{1,m_k}$; $l=\overline{1,N}$; $k=\overline{1,K}$).

6. Select a number of internal nodes located on the line segments between two points on the beams from stations to satellites.

7. Compute coordinates of internal nodes $\bar{x}_{ijpq}^{kr}$, $\bar{y}_{ijpq}^{kr}$, $\bar{z}_{ijpq}^{kr}$ (4) that lie on the segments between two points on the beams from stations to satellites ($r=\overline{1,M}$; $p,q\in[1,N]$; $i=\overline{1,n_k}$; $j=\overline{1,m_k}$; $k=\overline{1,K}$).

8. Using interpolation determine $\bar{v}_{ijpq}^{kr}$ (5) in the points defined in steps 6 and 7 ($i=\overline{1,n_k}$; $j=\overline{1,m_k}$; $p,q\in[1,N]$; $r=\overline{1,M}$; $k=\overline{1,K}$).

9. Determine boundaries $x_{\min}^{k}$, $y_{\min}^{k}$, $z_{\min}^{k}$, $x_{\max}^{k}$, $y_{\max}^{k}$, $z_{\max}^{k}$ (6) of the rectangular spatial area with the points $\bar{x}_{ijl}^{k}$, $\bar{y}_{ijl}^{k}$, $\bar{z}_{ijl}^{k}$ (2) and defined values $\bar{v}_{ijl}^{k}$ (3) ($i=\overline{1,n_k}$; $j=\overline{1,m_k}$; $l=\overline{1,N}$) for each measurement ($k=\overline{1,K}$).

10. Select a number of points L where the spatial area is divided and limited by the boundaries (6) set in step 9.

11. In the rectangular area defined in step 9 determine the coordinates of the equally located points $\hat{x}_i^k$, $\hat{y}_j^k$, $\hat{z}_l^k$ (7) ($i,j,l=\overline{1,L}$).

12. Join the sets of the nodes coordinates on the beams from stations to satellites (2) and on these beams (4) and sets of the correspondent known values (3), (5) into a combined set x_a^k, y_a^k, z_a^k, v_a^k (8), (9) ($a=\overline{1,A_k}$) that is experimentally defined discrete functional dependency of the ionization from spatial coordinates for a measurement k ($k=\overline{1,K}$).

13. Determine the best approximation basis common for all measurements $k=\overline{1,K}$ from the results of the problems (11), (12) obtained using the exponent reduction of the polynomial from numerous arguments (10) [3, 4].

14. Solve the problem (11), (12) for all measurements $k=\overline{1,K}$ using approximation basis determined in step 13.

15. Based on the results from step 14, using the interpolation by spline, define coefficient dependencies $c_i(t)$ ($i=\overline{1,n}$) of the polynomial (10) from time and

Conclusion

The resulting error indicators show that the developed algorithm **gives consistent results** for ionization field restoration that do not depend on the ionosphere state, satellites positions and changes in number of stations in the network used for computations. **Instant accuracy** of the ionization field restoration **is acceptable for our problem**.

To improve the described method, we need to conduct research to explain the structure of the approximating polynomial and search for **additional computation tools** to increase the approximation accuracy in the observational nodes and prevent rapid change of the approximating polynomial beyond these nodes.

The end!

Thanks!