PRECISE POINT POSITIONING FOR HIGH PRECISION GEODETIC APPLICATIONS

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1st International Conference:
NEW ADVANCED GNSS AND 3D SPATIAL TECHNIQUES
APPLICATIONS to CIVIL and ENVIRONMENTAL ENGINEERING, GEOPHYSICS,
ARCHITECTURE, ARCHEOLOGY and CULTURAL HERITAGE

Trieste, 18 – 20 February 2016
## Content of presentation

1. Introduction
2. PPP method
   - Geometrical aspect
   - Mathematical aspect
   - Uniqueness, (un)biasedness, estimability
3. Uniformity of PPP results
4. Case study
5. Summary
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- PPP – precise point positioning, middle of the 1990s
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- The necessities:
  - Proper set of unknowns (*coordinates*, *troposphere parameters*, *receiver clock errors*, *phase ambiguities*, *differential code biases*)
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  - Elimination/mitigation of GNSS biases to a mm level + cycle slips detection and removal
  - Highly precise/accurate IGS products
Development of PPP

- Content
- **Introduction**
- PPP principle
- Geometry of PPP
- GM model
- Uniqueness
- Unbiasedness
- Estimability
- Final results
- Uniformity
- Uniformity analysis
- Case study
- Summary
Development of PPP

- Multi-GNSS (GPS+GLONASS+Galileo+BDS+...)

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- Multi-GNSS (GPS+GLONASS+Galileo+BDS+...)
  - Additional unknowns – at least for receiver clock errors (differences between systems)
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Development of PPP

- Multi-GNSS (GPS+GLONASS+Galileo+BDS+...)
  - Additional unknowns – at least for receiver clock errors (differences between systems)

- Using raw observations instead of linear combinations

- Redefinition of unknowns – different receiver clock parameters for phase and code observations

- Ambiguity fixed PPP
  - Network solution only or
  - Additional external information
Principle of PPP

Receiver tracks signal from $k$ satellites (each epoch)

$L_1 = \rho(x, y, z) + \tau_R + T - I + N_1 + \xi_L + \varepsilon_{L_1}$

$L_2 = \rho(x, y, z) + \tau_R + T - \gamma I + N_2 + \xi_L + \varepsilon_{L_2}$

$P_1 = \rho(x, y, z) + \tau_R + T + I + D_1 + \xi_P + \varepsilon_{P_1}$

$P_2 = \rho(x, y, z) + \tau_R + T + \gamma I + D_2 + \xi_P + \varepsilon_{P_2}$
Principle of PPP

Receiver tracks signal from $k$ satellites (each epoch)

\[ L_1 = \rho(x, y, z) + \tau_R + T - I + N_1 + \xi_L + \varepsilon_{L_1} \]
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- Observations: $L_1$, $L_2$, $P_1$ and $P_2$ ([m])
Receiver tracks signal from $k$ satellites (each epoch)

\[ L_1 = \rho(x, y, z) + \tau_R + T - I + N_1 + \xi_L + \varepsilon_{L1} \]
\[ L_2 = \rho(x, y, z) + \tau_R + T - \gamma I + N_2 + \xi_L + \varepsilon_{L2} \]
\[ P_1 = \rho(x, y, z) + \tau_R + T + I + D_1 + \xi_P + \varepsilon_{P1} \]
\[ P_2 = \rho(x, y, z) + \tau_R + T + \gamma I + D_2 + \xi_P + \varepsilon_{P2} \]

- Observations: $L_1, L_2, P_1$ and $P_2$ ([m])
- Unknowns: $\rho(x, y, z), \tau_R, N_i, D_i$,
Principle of PPP

Receiver tracks signal from $k$ satellites (each epoch)

$L_1 = \rho(x, y, z) + \tau_R + I - N_1 + \xi_L + \varepsilon_{L_1}$

$L_2 = \rho(x, y, z) + \tau_R + T - \gamma I + N_2 + \xi_L + \varepsilon_{L_2}$

$P_1 = \rho(x, y, z) + \tau_R + T + \gamma I + D_1 + \xi_P + \varepsilon_{P_1}$

$P_2 = \rho(x, y, z) + \tau_R + T + I + D_2 + \xi_P + \varepsilon_{P_2}$

- Observations: $L_1$, $L_2$, $P_1$ and $P_2$ ([m])
- Unknowns: $\rho(x, y, z)$, $\tau_R$, $N_i$, $D_i$
- Biases:
  - models: $\xi_P$ and $\xi_L$
  - $\sim$approximation: $I$ and $T$
Principle of PPP

Receiver tracks signal from $k$ satellites (each epoch)

\begin{align*}
L_1 &= \rho(x, y, z) + \tau_R + T - I + N_1 + \xi_L + \varepsilon_{L_1} \\
L_2 &= \rho(x, y, z) + \tau_R + T - \gamma I + N_2 + \xi_L + \varepsilon_{L_2} \\
P_1 &= \rho(x, y, z) + \tau_R + T + I + D_1 + \xi_P + \varepsilon_{P_1} \\
P_2 &= \rho(x, y, z) + \tau_R + T + \gamma I + D_2 + \xi_P + \varepsilon_{P_2}
\end{align*}

- Observations: $L_1$, $L_2$, $P_1$ and $P_2$ ([m])
- Unknowns: $\rho(x, y, z)$, $\tau_R$, $N_i$, $D_i$
- Biases:
  - models: $\xi_P$ and $\xi_L$
  - $\sim$approximation: $I$ and $T$
- Random errors and unmodeled biases: $\varepsilon_i$
Principle of PPP

Preparing observation for processing:
Preprocessing for observation:

1. Modelling $\xi_P$ and $\xi_L$

2. \[ T = T_d + M_w T_w^z + M_g (G_N \cos \alpha + G_E \sin \alpha) \]

introduction of unknowns: $T_w^z, G_N$ in $G_E$
Principle of PPP

Preparing observation for processing:

1. Modelling $\xi_P$ and $\xi_L$

2. $T = T_d + M_w T_w^z + M_g (G_N \cos \alpha + G_E \sin \alpha)$
   introduction of unknowns: $T_w^z, G_N$ in $G_E$

3. Elimination of $I$ with linear combinations:

   \[
   L_3 = \rho + \tau_R + T_m + N_3 + \varepsilon_{L3}
   \]

   \[
   P_3 = \rho + \tau_R + T_m + D_3 + \varepsilon_{P3}
   \]
Principle of PPP

Preparing observation for processing:

1. Modelling $\xi_P$ and $\xi_L$

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3. Elimination of $I$ with linear combinations:
   
   $L_3 = \rho + \tau_R + T_m + N_3 + \varepsilon_{L3}$
   $P_3 = \rho + \tau_R + T_m + D_3 + \varepsilon_{P3}$

4. Setting up phase ambiguities (cycle slip determination)
Geometry of PPP
Geometry of PPP

\[ \rho(x, y, z) \quad T_m \quad \tau_R \quad N_3 \]

\[ S_1 \quad L_3^3 \quad \quad P_3 \]

\[ t_1 \quad S_2 \quad L_3^3 \quad \quad P_3 \]

\[ S_3 \quad L_3^3 \quad \quad P_3 \]

\[ \rho(x, y, z) \quad T_m \quad \tau_R \quad N_3 \]
Geometry of PPP

\[ \rho(x, y, z) \quad T_m \quad \tau_R \quad N_3 \]

\[ S_1 \quad \frac{L_3}{P_3} \]

\[ t_1 \quad S_2 \quad \frac{L_3}{P_3} \]

\[ S_3 \quad \frac{L_3}{P_3} \]

\[ S_1 \quad \frac{L_3}{P_3} \]

\[ t_2 \quad S_2 \quad \frac{L_3}{P_3} \]

\[ S_3 \quad \frac{L_3}{P_3} \]
Geometry of PPP

\[ \rho(x, y, z) T_m \tau_R N_3 \]

- \( S_1 \) \( L_3^3 \) \( P_3 \)
- \( t_1 \)
- \( S_2 \) \( L_3^3 \) \( P_3 \)
- \( S_3 \) \( L_3^3 \) \( P_3 \)
- \( \rho(x, y, z) \)
- \( T_m \)
- \( \tau_R \)
- \( N_3 \)
- \( D_3 \)

- \( t_2 \)
- \( S_2 \) \( L_3^3 \) \( P_3 \)
- \( S_3 \) \( L_3^3 \) \( P_3 \)

- \( t_3 \)
- \( S_3 \) \( L_3^3 \) \( P_3 \)
- \( S_2 \) \( L_3^3 \) \( P_3 \)
- \( S_1 \) \( L_3^3 \) \( P_3 \)
Geometry of PPP

\[ \rho(x, y, z) T_m \tau_R N_3 D_3 \]

- nonlinear
- variable \((S_i, t_i)\)

\[ \rho(x, y, z) \text{ and } T_m: \]
Geometry of PPP

\[ \rho(x, y, z) T_m \tau_R N_3 \]

- \( \rho(x, y, z) \) and \( T_m \):
  - nonlinear
  - variable \((S_i, t_i)\)
- \( \tau_R \):
  - linear
  - variable \((t_i)\)
Geometry of PPP

<table>
<thead>
<tr>
<th></th>
<th>$S_1 \frac{L_3}{P_3}$</th>
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<tr>
<td><strong>$t_1$</strong></td>
<td>$\rho(x, y, z) \ T_m \ \tau_R \ N_3 \ D_3$</td>
<td>$\rho(x, y, z)$ and $T_m$:</td>
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<td><strong>$t_2$</strong></td>
<td>$\rho(x, y, z) \ T_m \ \tau_R \ N_3 \ D_3$</td>
<td>$\tau_R$:</td>
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<td><strong>$t_3$</strong></td>
<td>$\rho(x, y, z) \ T_m \ \tau_R \ N_3 \ D_3$</td>
<td>$N_3$ and $D_3$:</td>
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- nonlinear
- variable ($S_i, t_i$)
- linear
- variable ($t_i$)
- linear
- variable ($S_i$)
Geometry of PPP

2 types of unknowns:
- nonlinear (left)
- linear (right)

Left unknowns:
- biased trilateration
- observable
Geometry of PPP

2 types of unknowns:
- nonlinear (left)
- linear (right)

Left unknowns:
- biased trilateration
- observable

Right unknowns:
Geometry of PPP

2 types of unknowns:
- nonlinear (left)
- linear (right)

Left unknowns:
- biased trilateration
- observable

Right unknowns:
Geometry of PPP

2 types of unknowns:
- nonlinear (left)
- linear (right)

Left unknowns:
- biased trilateration
- observable

Right unknowns:
- changeable
- observable? NO
Gauss-Markov model of PPP

Functional model – linearized observation equations

\[
E(L_3 - L_{3,0}) = \frac{\partial L_3}{\partial \rho(x, y, z)} \delta \rho + \frac{\partial L_3}{\partial T_m} \delta T_m + \frac{\partial L_3}{\partial \tau_R} \delta \tau_R + \frac{\partial L_3}{\partial N_3} \delta N_3
\]

\[
E(P_3 - P_{3,0}) = \frac{\partial P_3}{\partial \rho(x, y, z)} \delta \rho + \frac{\partial P_3}{\partial T_m} \delta T_m + \frac{\partial P_3}{\partial \tau_R} \delta \tau_R + \frac{\partial P_3}{\partial D_3} \delta D_3
\]
Gauss-Markov model of PPP

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\]

\[
E(L_3 - L_{3,0}) = \frac{\partial L_3}{\partial \rho(x, y, z)} \delta \rho + \frac{\partial L_3}{\partial T_m} \delta T_m + 1 \delta \tau_R + 1 \delta N_3
\]

\[
E(P_3 - P_{3,0}) = \frac{\partial P_3}{\partial \rho(x, y, z)} \delta \rho + \frac{\partial P_3}{\partial T_m} \delta T_m + 1 \delta \tau_R + 1 \delta D_3
\]
Gauss-Markov model of PPP

Functional model – linearized observation equations

\[
\begin{align*}
E(L_3 - L_{3,0}) &= \frac{\partial L_3}{\partial \rho(x, y, z)} \delta \rho + \frac{\partial L_3}{\partial T_m} \delta T_m + \frac{\partial L_3}{\partial \tau_R} \delta \tau_R + \frac{\partial L_3}{\partial N_3} \delta N_3 \\
E(P_3 - P_{3,0}) &= \frac{\partial P_3}{\partial \rho(x, y, z)} \delta \rho + \frac{\partial P_3}{\partial T_m} \delta T_m + \frac{\partial P_3}{\partial \tau_R} \delta \tau_R + \frac{\partial P_3}{\partial D_3} \delta D_3
\end{align*}
\]

All observed \( L_3 \) and \( P_3 \) from all satellites and all epochs into:

\[ E(f) = B \Delta \]
Gauss-Markov model of PPP

Functional model – linearized observation equations

\[
E(L_3 - L_{3,0}) = \frac{\partial L_3}{\partial \rho(x, y, z)} \delta \rho + \frac{\partial L_3}{\partial T_m} \delta T_m + \frac{\partial L_3}{\partial \tau_R} \delta \tau_R + \frac{\partial L_3}{\partial N_3} \delta N_3
\]

\[
E(P_3 - P_{3,0}) = \frac{\partial P_3}{\partial \rho(x, y, z)} \delta \rho + \frac{\partial P_3}{\partial T_m} \delta T_m + \frac{\partial P_3}{\partial \tau_R} \delta \tau_R + \frac{\partial P_3}{\partial D_3} \delta D_3
\]

All observed \( L_3 \) and \( P_3 \) from all satellites and all epochs into:

\[
E(f) = B \Delta
\]

Stochastic model – \( \sigma_{P_3} = 100 \sigma_{L_3}, p(e) = \cos(e) \)
Gauss-Markov model

Gauss-Markov (GM) linearized model

\[
E = \begin{bmatrix}
  f_{L,1} \\
  f_{P,1} \\
  \vdots \\
  f_{L,n} \\
  f_{P,n}
\end{bmatrix} = \begin{bmatrix}
  B_1^\rho & B_1^{T_m} & B_1^N & 0 & 1 & \cdots & 0 \\
  B_1^\rho & B_1^{T_m} & 0 & B_1^D & 1 & \cdots & 0 \\
  \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\
  B_n^\rho & B_n^{T_m} & B_n^N & 0 & 0 & \cdots & 1 \\
  B_n^\rho & B_n^{T_m} & 0 & B_n^D & 0 & \cdots & 1
\end{bmatrix}\begin{bmatrix}
  \Delta \rho \\
  \Delta T_m \\
  \Delta N \\
  \Delta D \\
  \delta \tau R,1 \\
  \vdots \\
  \tau R,n
\end{bmatrix}
\]
Gauss-Markov model

**Gauss-Markov (GM) linearized model**

\[
E \begin{bmatrix}
f_{L,1} \\
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\end{bmatrix} =
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\vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\
B_n^\rho & B_n^{T_m} & B_n^N & 0 & 0 & \cdots & 1 \\
B_n^\rho & B_n^{T_m} & 0 & B_n^D & 0 & \cdots & 1
\end{bmatrix}
\begin{bmatrix}
\Delta_\rho \\
\Delta_{T_m} \\
\Delta_N \\
\Delta_D \\
\delta \tau_{R,1} \\
\vdots \\
\tau_{R,n}
\end{bmatrix}
\]

**Solution to GM model:**

- **Consistency:**
  \[E(f) = B\Delta \rightarrow f = B\Delta + v \rightarrow v^T P v \rightarrow \text{min.}\]

- **Uniqueness:** \( r(B) = u \) (\( u \)-number of unknowns)
Uniqueness of Gauss-Markov model

- Content
- Introduction
- PPP principle
- Geometry of PPP
- GM model
- **Uniqueness**
- Unbiasedness
- Estimability
- Final results
- Uniformity
- Uniformity analysis
- Case study
- Summary
Uniqueness of Gauss-Markov model

- \( r(B) = u \implies By = 0 \) only when \( y = 0 \)
Uniqueness of Gauss-Markov model

- \( r(B) = u \quad \rightarrow \quad By = 0 \) only when \( y = 0 \)
- However:

\[
y_{u \times 1} = \begin{bmatrix} 0 \rho & 0_{T_m} & 1_N & 1_D & -1_{\tau_R} \end{bmatrix}^T
\]
Uniqueness of Gauss-Markov model

- \( r(B) = u \quad \rightarrow \quad By = 0 \) only when \( y = 0 \)
- However:
  \[
  y_{u \times 1} = \begin{bmatrix}
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  \end{bmatrix}^T
  \]
- We get: \( By = 0 \)
Uniqueness of Gauss-Markov model

- $r(B) = u \implies By = 0$ only when $y = 0$
- However:
  $$y_{u \times 1} = \begin{bmatrix} 0_{\rho} & 0_{Tm} & 1_N & 1_D & -1_{\tau_R} \end{bmatrix}^T$$
- We get: $By = 0$
- Matrix $B$ is singular
Uniqueness of Gauss-Markov model

- \( r(B) = u \Rightarrow By = 0 \) only when \( y = 0 \)
- However:
  \[
y_{u \times 1} = \begin{bmatrix} 0 & 0 \rho & 0_{T_m} & 1_N & 1_D & -1_{\tau_R} \end{bmatrix}^T
  \]
- We get: \( By = 0 \)
- Matrix \( B \) is singular
- Rank defect: \( d = u - r(B) = 1 \)
Uniqueness of Gauss-Markov model

- \( r(B) = u \quad \rightarrow \quad By = 0 \) only when \( y = 0 \)
- However:
  \[
  y_{u \times 1} = \begin{bmatrix}
  0_{ho} & 0_{Tm} & 1_N & 1_D & -1_{\tau_R}
  \end{bmatrix}^T
  \]
- We get: \( By = 0 \)
- Matrix \( B \) is singular
- Rank defect: \( d = u - r(B) = 1 \)
- \( y \) spans \( N(B) \)
Uniqueness of Gauss-Markov model

- \( r(B) = u \quad \rightarrow \quad By = 0 \) only when \( y = 0 \)
- However:
  \[
  y_{u \times 1} = \begin{bmatrix} 0 \rho & 0_{Tm} & 1_N & 1_D & -1_{\tau R} \end{bmatrix}^T
  \]
- We get: \( By = 0 \)
- Matrix \( B \) is singular
- Rank defect: \( d = u - r(B) = 1 \)
- \( y \) spans \( N(B) \)
- Solution to GM model: \( \Delta = \Delta_p + \gamma y \ (\gamma \in \mathbb{R}) \)
Uniqueness of Gauss-Markov model

- \( r(B) = u \rightarrow By = 0 \) only when \( y = 0 \)
- However:
  \[
  y_{u \times 1} = \begin{bmatrix}
  0 & 0 & T_m & 1_N & 1_D & -1_{\tau_R}
  \end{bmatrix}^T
  \]
- We get: \( By = 0 \)
- Matrix \( B \) is singular
- Rank defect: \( d = u - r(B) = 1 \)
- \( y \) spans \( N(B) \)
- Solution to GM model: \( \Delta = \Delta_p + \gamma y \) (\( \gamma \in \mathbb{R} \))
- Geometrical representation of \( N(B) \):
  \[
  \sum \delta N + \sum \delta D - \sum \delta \tau_R = 0
  \]
Unique solution to Gauss-Markov model

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Minimally constrained least squares solution
Unique solution to Gauss-Markov model

- Minimally constrained least squares solution
  - exactly $d = 1$ constraining equation
Unique solution to Gauss-Markov model

- Minimally constrained least squares solution
  - exactly $d = 1$ constraining equation
  - $v$ invariant to selection of constraining equation
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Unique solution to Gauss-Markov model

- Minimally constrained least squares solution
  - exactly $d = 1$ constraining equation
  - $v$ invariant to selection of constraining equation

- Constraining equation: $h^T \Delta = 0$ ($h = Ey$ and $E$ full rank)

- Particular solution:
  \[
  \Delta_h = (N + hh^T)^{-1} \cdot b \\
  Q_{\Delta_h} = (N + hh^T)^{-1} - y (y^T hh^T y)^{-1} y^T
  \]
Unique solution to Gauss-Markov model

- Minimally constrained least squares solution
  - exactly \( d = 1 \) constraining equation
  - \( v \) invariant to selection of constraining equation
- Constraining equation: \( h^T \Delta = 0 \) (\( h = Ey \) and \( E \) full rank)
- Particular solution:
  \[
  \Delta_h = (N + hh^T)^{-1} \cdot b
  \]
  \[
  Q\Delta_h = (N + hh^T)^{-1} - y (y^T hh^T y)^{-1} y^T
  \]
- Particular solution with \( w^T \Delta = 0 \) \( \rightarrow \) S-transformation
  \[
  S = I - y (w^T y)^{-1} w^T \quad \rightarrow \quad \Delta_w = S\Delta_h
  \]
  \[
  Q\Delta_w = SQ\Delta_h S^T
  \]
(Un)biasedness of estimated unknowns in PPP

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(Un)biasedness of estimated unknowns in PPP

- Singularity of $\mathbf{B}$: $\mathbb{E}(\Delta) = S\hat{\Delta} \neq \hat{\Delta}$
(Un)biasedness of estimated unknowns in PPP

- Singularity of $\mathbf{B}$: $\mathbf{E}(\Delta) = \mathbf{S}\hat{\Delta} \neq \hat{\Delta}$
- However:

$$
\mathbf{S} = \begin{bmatrix}
\mathbf{I}_\rho & 0 & 0 & 0 & 0 \\
0 & \mathbf{I}_{T_m} & 0 & 0 & 0 \\
0 & 0 & \Gamma_N & \Gamma_{ND} & \Gamma_{N\tau_R} \\
0 & 0 & \Gamma_{DN} & \Gamma_D & \Gamma_{D\tau_R} \\
0 & 0 & \Gamma_{\tau_R N} & \Gamma_{\tau_R D} & \Gamma_{\tau_R}
\end{bmatrix}
$$
(Un)biasedness of estimated unknowns in PPP

- Singularity of $\mathbf{B}$: $\mathbb{E}(\mathbf{\Delta}) = \mathbf{S}\mathbf{\hat{\Delta}} \neq \mathbf{\Delta}$
- However:
  \[
  \mathbf{S} = \begin{bmatrix}
  I_{\rho} & 0 & 0 & 0 & 0 \\
  0 & I_{T_m} & 0 & 0 & 0 \\
  0 & 0 & \Gamma_N & \Gamma_{ND} & \Gamma_{N\tau_R} \\
  0 & 0 & \Gamma_{DN} & \Gamma_D & \Gamma_{D\tau_R} \\
  0 & 0 & \Gamma_{\tau_R N} & \Gamma_{\tau_R D} & \Gamma_{\tau_R}
  \end{bmatrix}
  \]
- Consequently:
  \[
  \mathbb{E}(\mathbf{\Delta}) = \mathbf{S}\mathbf{\hat{\Delta}} \quad \rightarrow \quad 
  \begin{align*}
  \mathbb{E}(\mathbf{\Delta}_\rho) &= \mathbf{\hat{\Delta}}_\rho \\
  \mathbb{E}(\mathbf{\Delta}_{T_m}) &= \mathbf{\hat{\Delta}}_{T_m} \\
  \mathbb{E}(\mathbf{\Delta}_N) &= \Gamma_N\mathbf{\hat{\Delta}}_N + \Gamma_{ND}\mathbf{\hat{\Delta}}_D + \Gamma_{N\tau_R}\mathbf{\hat{\Delta}}_{\tau_R} \\
  \mathbb{E}(\mathbf{\Delta}_D) &= \Gamma_{DN}\mathbf{\hat{\Delta}}_N + \Gamma_D\mathbf{\hat{\Delta}}_D + \Gamma_{D\tau_R}\mathbf{\hat{\Delta}}_{\tau_R} \\
  \mathbb{E}(\mathbf{\Delta}_{\tau_R}) &= \Gamma_{\tau_R N}\mathbf{\hat{\Delta}}_N + \Gamma_{\tau_R D}\mathbf{\hat{\Delta}}_D + \Gamma_{\tau_R}\mathbf{\hat{\Delta}}_{\tau_R}
  \end{align*}
\]
(Un)biasedness of estimated unknowns in PPP

- Singularity of $B$: $E(\Delta) = S\hat{\Delta} \neq \hat{\Delta}$
- However:
  \[
  S = \begin{bmatrix}
    I_\rho & 0 & 0 & 0 & 0 \\
    0 & I_{Tm} & 0 & 0 & 0 \\
    0 & 0 & \Gamma_N & \Gamma_{ND} & \Gamma_{N\tau_R} \\
    0 & 0 & \Gamma_{DN} & \Gamma_D & \Gamma_{D\tau_R} \\
    0 & 0 & \Gamma_{\tau_RN} & \Gamma_{\tau_RD} & \Gamma_{\tau_R}
  \end{bmatrix}
  \]
- Consequently:
  \[
  E(\Delta) = S\hat{\Delta} \rightarrow \begin{align*}
  E(\Delta_\rho) &= \hat{\Delta}_\rho \\
  E(\Delta_{Tm}) &= \hat{\Delta}_{Tm} \\
  E(\Delta_N) &= \Gamma_N \hat{\Delta}_N + \Gamma_{ND} \hat{\Delta}_D + \Gamma_{N\tau_R} \hat{\Delta}_{\tau_R} \\
  E(\Delta_D) &= \Gamma_{DN} \hat{\Delta}_N + \Gamma_D \hat{\Delta}_D + \Gamma_{D\tau_R} \hat{\Delta}_{\tau_R} \\
  E(\Delta_{\tau_R}) &= \Gamma_{\tau_RN} \hat{\Delta}_N + \Gamma_{\tau_RD} \hat{\Delta}_D + \Gamma_{\tau_R} \hat{\Delta}_{\tau_R}
  \end{align*}
  \]
- Unbiased unknowns: $x, y, z, T_m$
- Biased unknowns: $N, D, \tau_R$
Estimable unknowns of PPP

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Estimable unknowns of PPP

- \( r(B) = u - 1 < u \rightarrow \text{unknowns are NOT estimable} \)
Estimable unknowns of PPP

- $r(B) = u - 1 \leq u \rightarrow$ unknowns are NOT estimable
- Number of estimable unknowns: $r(B)$
Estimable unknowns of PPP

- $r(B) = u - 1 < u \rightarrow$ unknowns are NOT estimable
- Number of estimable unknowns: $r(B)$
- However:

$$
\begin{bmatrix}
\Delta^*_N \\
\Delta^*_D \\
\Delta^*_\tau_R
\end{bmatrix}
= 
\begin{bmatrix}
I_N & 0 & 1_N & 0 \\
0 & I_D & 1_D & 0 \\
0 & 0 & -1^\tau_R -1 & I_{(\tau_R - 1) \times \tau_R}
\end{bmatrix}
\begin{bmatrix}
\Delta_N \\
\Delta_D \\
\delta\tau_{R,1} \\
\Delta_{\tau_R,1}
\end{bmatrix}
$$
Estimable unknowns of PPP

- \( r(B) = u - 1 < u \) → unknowns are NOT estimable
- Number of estimable unknowns: \( r(B) \)
- However:
  \[
  \begin{bmatrix}
  \Delta^*_N \\
  \Delta^*_D \\
  \Delta^*_{\tau_R}
  \end{bmatrix} =
  \begin{bmatrix}
  I_N & 0 & 1_N & 0 \\
  0 & I_D & 1_D & 0 \\
  0 & 0 & -1_{\tau_R-1} & I_{(\tau_R-1)\times\tau_R}
  \end{bmatrix}
  \begin{bmatrix}
  \Delta_N \\
  \Delta_D \\
  \delta\tau_{R,1} \\
  \Delta_{\tau_R,1}
  \end{bmatrix}
  \]
- Estimable unknowns are:
Estimable unknowns of PPP

- $r(B) = u - 1 < u \rightarrow$ unknowns are NOT estimable
- Number of estimable unknowns: $r(B)$
- However:
  \[
  \begin{bmatrix}
  \Delta_{N}^* \\
  \Delta_{D}^* \\
  \Delta_{\tau R}^*
  \end{bmatrix} =
  \begin{bmatrix}
  I_N & 0 & 1_N & 0 \\
  0 & I_D & 1_D & 0 \\
  0 & 0 & -1_{\tau R-1} & I_{(\tau R-1) \times \tau R}
  \end{bmatrix}
  \begin{bmatrix}
  \Delta_N \\
  \Delta_D \\
  \delta_{\tau R,1} \\
  \Delta_{\tau R,1}
  \end{bmatrix}
  \]
- Estimable unknowns are:
  - $x, y, z, T_m$
  - $\Delta_{N}^* = \Delta_N + \delta_{\tau R,1}$
  - $\Delta_{D}^* = \Delta_D + \delta_{\tau R,1}$
  - $\Delta_{\tau R}^* = \Delta_{\tau R,1} - \delta_{\tau R,1}$
Final results of PPP

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Final results of PPP

- One-step solution of GM model inconvenient (too large to process)
Final results of PPP

- One-step solution of GM model inconvenient (too large to process)
- Procedure:
Final results of PPP

- One-step solution of GM model inconvenient (too large to process)

- Procedure:
  - Elimination of receiver clock error from GM model for each epoch
  - Stacking of epoch normal equations
Final results of PPP

- One-step solution of GM model inconvenient (too large to process)

- Procedure:
  - Elimination of receiver clock error from GM model for each epoch
  - Stacking of epoch normal equations

- Unknowns: $\Delta \rho$, $\Delta T_m$, $\Delta N$, $\Delta D$
Final results of PPP

- One-step solution of GM model inconvenient (too large to process)

- Procedure:
  - Elimination of receiver clock error from GM model for each epoch
  - Stacking of epoch normal equations

- Unknowns: $\Delta \rho, \Delta T_m, \Delta N, \Delta D$

- Constraining equation: $\sum D = 0$
Final results of PPP

- One-step solution of GM model inconvenient (too large to process)

- Procedure:
  - Elimination of receiver clock error from GM model for each epoch
  - Stacking of epoch normal equations

- Unknowns: $\Delta \rho$, $\Delta T_m$, $\Delta N$, $\Delta D$

- Constraining equation: $\sum D = 0$

- Estimation of $\tau_R$ and $\sigma^2_{\tau_R}$ subsequently
Final results of PPP

- One-step solution of GM model inconvenient (too large to process)

- Procedure:
  - Elimination of receiver clock error from GM model for each epoch
  - Stacking of epoch normal equations

- Unknowns: $\Delta \rho$, $\Delta T_m$, $\Delta N$, $\Delta D$

- Constraining equation: $\sum D = 0$

- Estimation of $\tau_R$ and $\sigma_{\tau_R}^2$ subsequently

- Consequence:
Final results of PPP

- One-step solution of GM model inconvenient (too large to process)

- Procedure:
  - Elimination of receiver clock error from GM model for each epoch
  - Stacking of epoch normal equations

- Unknowns: \( \Delta \rho, \Delta T_m, \Delta N, \Delta D \)

- Constraining equation: \( \sum D = 0 \)

- Estimation of \( \tau_R \) and \( \sigma^2_{\tau_R} \) subsequently

- Consequence:
  - Calculation of any \( \Delta w \) possible
  - Calculation of \( Q_{\Delta w} \) NOT possible
Uniformity of PPP coordinates

PPP survey on a set of new geodetic points
Uniformity of PPP coordinates

Estimated PPP coordinates in a global coordinate system
Uniformity of PPP coordinates

Quality control - reference stations are included in survey (known coordinates)
Uniformity of PPP coordinates

Estimated PPP coordinates of all geodetic points in global coordinate system
Uniformity of PPP coordinates

Differences of PPP coordinates with respect to reference coordinates
Uniformity of PPP coordinates

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Uniformity of PPP coordinates

- Differences $X^P - X^I$ contain:
Uniformity of PPP coordinates

- Differences $X^P - X^I$ contain:
  - Random component and
  - (Also) systematic component (bias)
Uniformity of PPP coordinates

- Differences $X^P - X^I$ contain:
  - Random component and
  - (Also) systematic component (bias)

- Sources of differences $X^P - X^I$:

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Uniformity of PPP coordinates

- Differences $X^P - X^I$ contain:
  - Random component and
  - (Also) systematic component (bias)

- Sources of differences $X^P - X^I$:
  - Coordinate system, processing algorithm, errors in PPP models/processing...
Uniformity of PPP coordinates

- Differences $\mathbf{X}^P - \mathbf{X}^I$ contain:
  - Random component and
  - (Also) systematic component (bias)

- Sources of differences $\mathbf{X}^P - \mathbf{X}^I$:
  - Coordinate system, processing algorithm, errors in PPP models/processing...

- Removal of systematic component:
Uniformity of PPP coordinates

- Differences $X^P - X^I$ contain:
  - Random component and
  - (Also) systematic component (bias)

- Sources of differences $X^P - X^I$:
  - Coordinate system, processing algorithm, errors in PPP models/processing...

- Removal of systematic component:
  - spatial similarity transformation
  - presumption: small values of transformation parameters

...representation of PPP coordinates in uniform global coordinate system.
Similarity transformation

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Similarity transformation

- Functional model:

\[ X^P = X^I + T + mX^I + R(\omega_x, \omega_y, \omega_z)X^I \]
Similarity transformation

- Functional model:

\[ X^P = X^I + T + mX^I + R(\omega_x, \omega_y, \omega_z)X^I \]

- for each point \( i \in \{1, \ldots, n_p\} \):
Similarity transformation

- Functional model:
  \[ X^P = X^I + T + mX^I + R(\omega_x, \omega_y, \omega_z)X^I \]

  - for each point \( i \in \{1, \ldots, n_p\} \):
    - PPP results: \( X^P_i \) in \( \Sigma^P_i \),
Similarity transformation

- Functional model:
  \[ X^P = X^I + T + mX^I + R(\omega_x, \omega_y, \omega_z)X^I \]

- for each point \( i \in \{1, \ldots, n_p\} \):
  - PPP results: \( X^P_i \) in \( \Sigma^P_i \),
  - Functional model: \( v_i + M_i \beta = f_i = X^I_i - X^P_i \)
Similarity transformation

- Functional model:
  \[ X^P = X^I + T + mX^I + R(\omega_x, \omega_y, \omega_z)X^I \]

- for each point \( i \in \{1, \ldots, n_p\} \):
  - PPP results: \( X^P_i \) in \( \Sigma^P_i \),
  - Functional model: \( v_i + M_i \beta = f_i = X^I_i - X^P_i \)
  - Stochastic model: \( P_i = \delta \cdot (\Sigma^P_i)^{-1} \quad \delta \in \{0, 1\} \)
Similarity transformation

- Functional model:
  \[ X^P = X^I + T + mX^I + R(\omega_x, \omega_y, \omega_z)X^I \]

- for each point \( i \in \{1, \ldots, n_p\} \):
  - PPP results: \( X^P_i \) in \( \Sigma^P_i \),
  - Functional model: \( v_i + M_i\beta = f_i = X^I_i - X^P_i \)
  - Stochastic model: \( P_i = \delta \cdot (\Sigma^P_i)^{-1} \quad \delta \in \{0, 1\} \)

- Final mathematical model:
  \[ v + M\beta = f \quad P = \begin{bmatrix} P_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & P_{n_p} \end{bmatrix} \]

- Matrix \( M \): the inner constraints matrix
Similarity transformation

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Similarity transformation

- Least squares estimation

\[ \beta = (M^T P M)^{-1} M^T P f \]

\[ Q_\beta = (M^T P M)^{-1} \]

\[ v = f - M\beta = (I - MQ_\beta M^T P) f \]
Similarity transformation

- Least squares estimation

\[ \beta = (M^T P M)^{-1} M^T P f \]
\[ Q_\beta = (M^T P M)^{-1} \]
\[ v = f - M \beta = (I - M Q_\beta M^T P) f \]

- Transformed PPP coordinates \( X^t \):

\[ X^t = X^P + M\beta = X^I - v \]
Similarity transformation

- Least squares estimation
  \[ \beta = (M^T P M)^{-1} M^T P f \quad Q_\beta = (M^T P M)^{-1} \]
  \[ v = f - M \beta = (I - MQ_\beta M^T P) f \]

- Transformed PPP coordinates \( X^t \):
  \[ X^t = X^P + M \beta = X^I - v \]

- Precision of transformed coordinates \( Q_{X^t} \):
  \[ Q_{X^t} = Q_v = (I - MQ_\beta M^T P) Q^P (I - MQ_\beta M^T P)^T \]
Similarity transformation

- Least squares estimation
  \[ \beta = (M^T P M)^{-1} M^T P f \]
  \[ Q_\beta = (M^T P M)^{-1} \]
  \[ v = f - M\beta = (I - MQ_\beta M^T P) f \]

- Transformed PPP coordinates \( X^t \):
  \[ X^t = X^P + M\beta = X^I - v \]

- Precision of transformed coordinates \( Q_{X^t} \):
  \[ Q_{X^t} = Q_v = (I - MQ_\beta M^T P) Q^P (I - MQ_\beta M^T P)^T \]

- Let’s define \( W^T = M^T P \) and therefore:
  \[ S = (I - MQ_\beta M^T P) = (I - M (W^T M)^{-1} W^T) \]
Similarity transformation

- Least squares estimation
  \[ \beta = (M^T P M)^{-1} M^T P f \]
  \[ Q_{\beta} = (M^T P M)^{-1} \]
  \[ v = f - M\beta = (I - MQ_{\beta} M^T P) f \]

- Transformed PPP coordinates \( X^t \):
  \[ X^t = X^P + M\beta = X^I - v \]

- Precision of transformed coordinates \( Q_{X^t} \):
  \[ Q_{X^t} = Q_v = (I - MQ_{\beta} M^T P) Q^P (I - MQ_{\beta} M^T P)^T \]

- Let’s define \( W^T = M^T P \) and therefore:
  \[ S = (I - MQ_{\beta} M^T P) = (I - M (W^T M)^{-1} W^T) \]

- We get:
  \[ v = Sf \]
  \[ Q_v = SQ^P S^T \]

S-TRANSFORMATION
Similarity transformation results analysis

- Obtained solution: $X_t$ in $\Sigma X_t$
Similarity transformation results analysis

- Obtained solution: \( X^t \) in \( \Sigma X^t \)
- May be described with S-transformation: \( \Sigma X^t = S \Sigma P S^T \)
Similarity transformation results analysis

- Obtained solution: $X^t$ in $\Sigma_{X^t}$
- May be described with S-transformation: $\Sigma_{X^t} = S\Sigma^P S^T$
- Properties of S-transformation:
  - $S$ idempotent
  - $r(S) = 3n_p - u_t$, ($u_t$ number of transformation parameters)
  - $r(\Sigma_{X^t}) = r(S)$
  - $N(\Sigma_{X^t}) = N(S) = M$ and $R(\Sigma_{X^t}) = R(S) = M^\perp$
  - $3n_p = r(\Sigma^P) \geq r(\Sigma_{X^t}) = 3n_p - u_t \geq 3n_p - 7$
**Similarity transformation results analysis**

- Obtained solution: \( X^t \) in \( \Sigma_{X^t} \)
- May be described with S-transformation: \( \Sigma_{X^t} = S \Sigma^P S^T \)
- Properties of S-transformation:
  - \( S \) idempotent
  - \( r(S) = 3n_p - u_t \), \( u_t \) number of transformation parameters
  - \( r(\Sigma_{X^t}) = r(S) \)
  - \( N(\Sigma_{X^t}) = N(S) = M \) and \( R(\Sigma_{X^t}) = R(S) = M^\perp \)
  - \( 3n_p = r(\Sigma^P) \geq r(\Sigma_{X^t}) = 3n_p - u_t \geq 3n_p - 7 \)
- S-transformation may be used to change (reduce) rank of covariance matrix
Uniform PPP results

All points obtain a correction after transformation
Uniform PPP results

Transformed coordinates are congruent to uniform global system
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GPS data from permanent station in/around Slovenia

<table>
<thead>
<tr>
<th>Network</th>
<th>#Stations</th>
<th>#Files</th>
<th>Time span</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIGNAL</td>
<td>16</td>
<td>24 457</td>
<td>2002–2013</td>
</tr>
<tr>
<td>FReDNet</td>
<td>14</td>
<td>36 165</td>
<td>2002–2013</td>
</tr>
<tr>
<td>APOS</td>
<td>8</td>
<td>11 531</td>
<td>2003–2010</td>
</tr>
<tr>
<td>EPN</td>
<td>5</td>
<td>7 943</td>
<td>2002–2013</td>
</tr>
<tr>
<td>IGS</td>
<td>18</td>
<td>71 696</td>
<td>2000–2013</td>
</tr>
<tr>
<td>ALL</td>
<td>63</td>
<td>153 792</td>
<td>2000–2013</td>
</tr>
</tbody>
</table>
Permanent stations in Europe
Permanent stations in/around Slovenia
Daily solution of PPP unknowns

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Daily solution of PPP unknowns

- GPS data only
Daily solution of PPP unknowns

- GPS data only
- Processing of all daily files – day by day
Daily solution of PPP unknowns

- GPS data only
- Processing of all daily files – day by day
- Estimated unknowns for each station (each day):
  - coordinates $X^P$
  - 13 zenith troposphere parameters $T^z_m$ (every 2h)
  - 4 troposphere gradients ($2 \times G_N$ and $2 \times G_E$)
  - $\tau_R$, $N$, $D$
Daily solution of PPP unknowns

- GPS data only
- Processing of all daily files – day by day
- Estimated unknowns for each station (each day):
  - coordinates $X^P$
  - 13 zenith troposphere parameters $T^z_m$ (every 2h)
  - 4 troposphere gradients ($2 \times G_N \text{ and } 2 \times G_E$)
  - $\tau_R, N, D$
- Coordinates estimated in coordinate system of precise ephemeris
Daily solution of PPP unknowns

- GPS data only
- Processing of all daily files – day by day
- Estimated unknowns for each station (each day):
  - coordinates $X^P$
  - 13 zenith troposphere parameters $T^z_m$ (every 2h)
  - 4 troposphere gradients ($2 \times G_N$ and $2 \times G_E$)
  - $\tau_R, N, D$
- Coordinates estimated in coordinate system of precise ephemeris
- Time series obtained for each station and each coordinate
Time series of daily coordinates
Repeatability of daily PPP coordinates

<table>
<thead>
<tr>
<th>Network</th>
<th>$\bar{\sigma}_N$ [mm]</th>
<th>$\bar{\sigma}_E$ [mm]</th>
<th>$\bar{\sigma}_U$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIGNAL</td>
<td>4,4</td>
<td>5,9</td>
<td>10,2</td>
</tr>
<tr>
<td>FReDNet</td>
<td>4,2</td>
<td>5,7</td>
<td>9,5</td>
</tr>
<tr>
<td>APOS</td>
<td>4,4</td>
<td>5,5</td>
<td>10,8</td>
</tr>
<tr>
<td>EPN</td>
<td>4,5</td>
<td>5,6</td>
<td>9,9</td>
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<tr>
<td>IGS</td>
<td>6,2</td>
<td>6,8</td>
<td>11,8</td>
</tr>
<tr>
<td><strong>All</strong></td>
<td><strong>4,8</strong></td>
<td><strong>6,0</strong></td>
<td><strong>10,6</strong></td>
</tr>
</tbody>
</table>

- sub cm level of repeatability for $N$ and $E$
- cm level of repeatability for $U$
- evident bias in coordinate time series
Transformation of PPP coordinates

- Content
- Introduction
- PPP principle
- Geometry of PPP
- GM model
- Uniqueness
- Unbiasedness
- Estimability
- Final results
- Uniformity
- Uniformity analysis
- Case study
- Summary
Transformation of PPP coordinates

- Reference coordinate system – IGb08
Transformation of PPP coordinates

- Reference coordinate system – IGb08
- 11 reference stations – IGS network
Transformation of PPP coordinates

- Reference coordinate system – IGb08
- 11 reference stations – IGS network
- 4 different transformations:
Transformation of PPP coordinates

- Reference coordinate system – IGb08
- 11 reference stations – IGS network
- 4 different transformations:
  - 3-parametric: $t_x, t_y, t_z$
  - 4-parametric: $t_x, t_y, t_z, m$
  - 6-parametric: $t_x, t_y, t_z, \omega_x, \omega_y, \omega_z$
  - 7-parametric: $t_x, t_y, t_z, \omega_x, \omega_y, \omega_z, m$
Transformation of PPP coordinates

- Reference coordinate system – IGb08
- 11 reference stations – IGS network
- 4 different transformations:
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  - 7-parametric: $t_x, t_y, t_z, \omega_x, \omega_y, \omega_z, m$

<table>
<thead>
<tr>
<th>Transformacija</th>
<th>$\bar{\sigma}_N$ [mm]</th>
<th>$\bar{\sigma}_E$ [mm]</th>
<th>$\bar{\sigma}_U$ [mm]</th>
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</thead>
<tbody>
<tr>
<td>3-parametric</td>
<td>3,1</td>
<td>5,1</td>
<td>7,8</td>
</tr>
<tr>
<td>4-parametric</td>
<td>3,1</td>
<td>4,9</td>
<td>7,8</td>
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<tr>
<td>6-parametric</td>
<td>3,1</td>
<td>5,0</td>
<td>6,4</td>
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<tr>
<td>7-parametric</td>
<td>3,1</td>
<td>4,8</td>
<td>6,4</td>
</tr>
</tbody>
</table>
Estimated transformation parameters

\[ t_x \ [mm] \]

\[ t_y \ [mm] \]

\[ t_z \ [mm] \]

\[ \omega_x \ [m''/y] \]

\[ \omega_y \ [m''/y] \]

\[ \omega_z \ [m''/y] \]

\[ m \ [ppb] \]
Impact of transf. parameters on coordinate repeatability

![Graph showing the impact of transf. parameters on coordinate repeatability](image-url)
Transformed time series of daily coordinates
Original time series of daily coordinates

- METS
- ZIMM
- GSR1
- ACOM

\[ \Delta U+50 \text{ [mm]} \]
\[ \Delta E \text{ [mm]} \]
\[ \Delta N-50 \text{ [mm]} \]
Impact of transf. parameters on coordinate repeatability

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# Impact of transf. parameters on coordinate repeatability

- # of parameters has no effect
Impact of transf. parameters on coordinate repeatability

- # of parameters has no effect
- 3-parametric transformation is sufficient
Impact of transf. parameters on coordinate repeatability

- # of parameters has no effect
- 3-parametric transformation is sufficient
- Results are theoretically and practically equivalent to differential positioning

<table>
<thead>
<tr>
<th></th>
<th>$\bar{\sigma}_N$ [mm]</th>
<th>$\bar{\sigma}_E$ [mm]</th>
<th>$\bar{\sigma}_U$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPP</td>
<td>2,9</td>
<td>4,2</td>
<td>7,0</td>
</tr>
<tr>
<td>Bernese</td>
<td>2,9</td>
<td>2,8</td>
<td>6,3</td>
</tr>
</tbody>
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Impact of transf. parameters on coordinate repeatability

- # of parameters has no effect
- 3-parametric transformation is sufficient
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<th>(\bar{\sigma}_E) [mm]</th>
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- Comparison to reference values

<table>
<thead>
<tr>
<th></th>
<th>(\sigma_N) [mm]</th>
<th>(\sigma_E) [mm]</th>
<th>(\sigma_U) [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPP–REF</td>
<td>1,3</td>
<td>1,6</td>
<td>2,9</td>
</tr>
<tr>
<td>Summary</td>
<td></td>
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<tr>
<td>Precise point positioning:</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>• Analitical description of mathematical model (singularity, null space)</td>
<td></td>
<td></td>
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<td>• Estimability of unknowns</td>
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Precise point positioning:

- Analytical description of mathematical model (singularity, null space)
- Estimability of unknowns
- (Un)biasedness of estimated unknowns

Uniformity of PPP results:

- Transformation represented with S-transformation
- Singularity of $\Sigma X^t$
- Generalization of S-transformation to reduce rank
Summary

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**Summary**

**Case study:**

- Repeatability of PPP coordinates on cm level
- Evident bias in time series of PPP coordinates
- 3-parametric transformation sufficient
- Repeatability of transformed PPP coordinates on a few mm level
- Comparable to differential techniques (Bernese)
- Theoretical and practical equivalence of PPP results
Thank you for your attention