Accuracy and Availability of the Real Time GNSS Geodetic Networks

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Global Navigation Satellite Systems Today

Autonomous:
- GPS (USA)
- GLONASS (Russia)
- Galileo (Europe)
- Compass (China)

Augmentation:
- **DGPS** – Maritime DGPS (aprox. - 300 stations), accuracy: 1 m (H), 2 m (V)
- **EGNOS, WAAS, MBAS** – geostationary satellite, accuracy: 3 m (H), 4 m (V)
- **Active Geodetic Network** – national and private reference networks accuracy: 1 cm (H), 2 cm (V)

**Past:** Positioning and Time services

**Future:** Positioning (few levels-services), Time Service, Search and Rescue Service, Safety of Live Service
Polish Active Geodetic Network as a part of EUPOS

**EUPOS**
- Central and Eastern Europe, 16 Nations
- Network of GNSS Reference stations
- Accuracy:
  - geodesy 1 cm (H); 2 cm (V)
  - navigation 2 cm (H); 3 cm (V)

**ASG-EUPOS ARCHITECTURE (2008)**
- New GPS reference stations – 68
- New GPS/GLONASS reference stations – 8
- Existing GNSS stations – 16
- Control station: Warszawa
- Secondary Control Station: Katowice
ASG-EUPOS services

GNSS Geodetic Networks have typically 3 types of services:

- Precise Postprocessing for Geodesy (RINEX)
- Precise Networking Real Time RTK for Geodesy and Navigation: RTCM 2.3 VRS, RTCM 2.3 FKP, RTCM VRS 3.0/3.1
- Real Time Differential GNSS for Navigation: RTCM 2.1

<table>
<thead>
<tr>
<th>Service</th>
<th>Method</th>
<th>Transmission</th>
<th>Accuracy</th>
<th>Format</th>
<th>Receiver</th>
</tr>
</thead>
<tbody>
<tr>
<td>NAVGEO</td>
<td>RTK</td>
<td>GSM/GPRS/EDGE/UMTS, Internet,</td>
<td>0.03m horizontal, 0.05m vertical</td>
<td>RTCM SC-104 version 2.3 and 3.0</td>
<td>L1/L2</td>
</tr>
<tr>
<td>CODGIS</td>
<td>DGPS</td>
<td>GSM/GPRS/EDGE/UMTS, Internet</td>
<td>up to 0.5m</td>
<td>RTCM SC-104 version 2.1</td>
<td>L1</td>
</tr>
<tr>
<td>NAVGIS</td>
<td>DGPS</td>
<td>GSM/GPRS/EDGE/UMTS, Internet, FM(optional)</td>
<td>1m - 3m</td>
<td>RTCM SC-104 version 2.1</td>
<td>L1</td>
</tr>
</tbody>
</table>

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</tr>
</thead>
<tbody>
<tr>
<td>POSGEO</td>
<td>postprocessing</td>
<td>Internet, CD-ROM</td>
<td>0.01m horizontal, 0.03m vertical</td>
<td>RINEX 2.x</td>
<td>L1, L1/L2</td>
</tr>
<tr>
<td>POSGEO D</td>
<td>postprocessing</td>
<td>Internet, CD-ROM</td>
<td>RINEX 2.x</td>
<td>L1, L1/L2</td>
<td></td>
</tr>
</tbody>
</table>
Main problems connected with railway geometrical forming systems are difficulties in **survey of the whole system in one consistent geodetic system**.

Line sections and curves of the railways are very often **so large that visual evaluation of their shape is impossible**.

Conventional geodesy requires **division of the railway route into smaller examined separately parts**.

It becomes an additional errors source and a total appraisal of particular geometrical system is difficult.

Development of geodesic satellite techniques combined with improving precision of measure GPS (Global Positioning System) induces geodesists to make an attempt to **apply GNSS networking solution techniques** for the purpose of making inventory of railway tracks.

Active Geodetic Networks allowed to determine **precise GNSS position based on GPS, GPS/GLONASS, GPS/GLONASS/BEIDOU in real time**.
Position accuracy measures – classical approach

The positioning accuracy means the degree to which the statistics of the measured positions of coordinates are consistent with the actual, real values, or those, which we take for real. A measure of the positioning accuracy is its error.

1. To record NMEA messages

2. Gauss-Kruger transformation from B, L [deg.] to x, y [m]

3. By the formulas:

\[
x = k \cdot R \cdot \left[ \frac{S(B)}{R} + \frac{(\Delta L)^2}{2} \cdot \sin(B) \cdot \cos(B) + \frac{(\Delta L)^4}{24} \cdot \sin(B) \cdot \cos^3(B) \cdot (5 - t^2 + 9 \cdot \eta^2 + 4 \cdot \eta^4) + \frac{(\Delta L)^6}{720} \cdot \sin(B) \cdot \cos^5(B) \cdot (61 - 58 \cdot t^2 + t^4 + 270 \cdot \eta^2 - 330 \cdot \eta^2 \cdot t^2 + 445 \cdot \eta^4) \right]
\]

\[
y = R \cdot \left[ \Delta L \cdot \cos(B) + \frac{(\Delta L)^3}{6} \cdot \cos^3(B) \cdot (1 - t^2 + \eta^2) + \frac{(\Delta L)^5}{120} \cdot \cos^5(B) \cdot (5 - 18 \cdot t^2 + t^4 + 14 \cdot \eta^2 - 58 \cdot \eta^2 \cdot t^2 + 13 \cdot \eta^4) \right]
\]

where:
- \(B, L\) – measured ellipsoidal coordinates;
- \(R\) – radius of curvature in the prime vertical;
- \(S(B)\) – distance from the Equator to defined coordinate \(B\) [m];
- \(\Delta L\) – difference in longitude between \(L\) and Prime Meridian [m];
- \(k\) – scale factor of 0.999923,
Position accuracy measures – classical approach

4. To calculate the measures:

<table>
<thead>
<tr>
<th>The measure of accuracy</th>
<th>Dimension</th>
<th>Probability</th>
<th>Formula</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS</td>
<td>1D</td>
<td>68.3%</td>
<td>$\text{RMS} = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \mu)^2}$</td>
<td>The standard deviation of the mean square error, which is relative to $\phi$, $\lambda$ or h</td>
</tr>
<tr>
<td>DRMS</td>
<td>2D</td>
<td>63.2-68.3%</td>
<td>$\text{DRMS}(2D) = \sqrt{\sigma_x^2 + \sigma_y^2}$</td>
<td>The root mean square error sum of squares, which is relative to $\phi$, $\lambda$, $h$</td>
</tr>
<tr>
<td>DRMS</td>
<td>3D</td>
<td>95.4-98.2%</td>
<td>$\text{DRMS}(3D) = 2\sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_h^2}$</td>
<td>Twice the DRMS (2DRMS)</td>
</tr>
<tr>
<td>CEP</td>
<td>2D</td>
<td>50%</td>
<td>$\text{CEP} = 0.589 \cdot (\sigma_x + \sigma_y)$</td>
<td>The radius of circle centered at the true position, containing the position estimate with probability of 50%</td>
</tr>
<tr>
<td>SEP</td>
<td>3D</td>
<td>50%</td>
<td>$\text{SEP} = 0.513 \cdot (\sigma_x + \sigma_y + \sigma_h)$</td>
<td>The radius of sphere centered at the true position, containing the position estimate with probability of 50%</td>
</tr>
<tr>
<td>R68</td>
<td>2D</td>
<td>68%</td>
<td>$\text{R68}(2D) = 1.28 \cdot \text{CEP}$</td>
<td>The radius of circle (sphere) centered at the true position, containing the position estimate with probability of 68%</td>
</tr>
<tr>
<td>R95</td>
<td>2D</td>
<td>95%</td>
<td>$\text{R95}(2D) = 2.08 \cdot \text{CEP}$</td>
<td>The radius of circle (sphere) centered at the true position, containing the position estimate with probability of 95%</td>
</tr>
</tbody>
</table>

Where: $\sigma_x$ – standard deviation of geodetic (geographical) latitude
$\sigma_y$ – standard deviation of geodetic (geographical) longitude
$\sigma_h$ – standard deviation of ellipsoidal height

5. To draw a few functions:

Example: DGPS horizontal position error distribution function calculated for 3 campaigns (number of fixes: 1-3 mln/campaign)

Example: Probability density function (Rayleigh distribution) for position error of DGPS for 3 campaigns.
Position accuracy measures – classical approach

... or in other forms:

GPS positions (aprox. 1.12 mln fixes)

DGPS positions (2.3 mln fixes)
### Availability definition

**System Availability** – probability that the system will be in working state in any moment of time (positioning, surveying, transmission, etc)

\[
\alpha(t) = \begin{cases} 
1, & Z_n^" \leq t < Z_{n+1}' \\
0, & Z_{n+1}' \leq t < Z_{n+1}" 
\end{cases}
\]

State of the system:

- \(X_1, X_2, \ldots\) working times
- \(Y_1, Y_2, \ldots\) times of failures
- \(Z_n^" = X_1 + Y_1 + X_2 + Y_2 + \ldots + Y_{n-1} + X_n\) moments of failures
- \(Z_n' = Z_n'' + Y_n\) moments of renewal

\[
A = \frac{E(X)}{E(X) + E(Y)} = \frac{MTBF}{MTBF + MTTR}
\]

Example: Transmission availability zones of the Polish DGPS Reference station Rozewie (Algorithms: C. Specht, software A. Nowak)
Let the GNSS networking position error (3D, 2D, 1D) is variable as a function of time.

We can arbitrary define value – maximum acceptable position error (for example – defined in Radionavigational Plan) for which the availability will be measured.

We can recognize two states: the working state - where the error is lower than acceptable value and failure state - where the error is higher.

**Working state**

Position error lower than maximum acceptable position error.

**Failure state**

Position error is higher than maximum acceptable position error.
Positioning GNSS networking service availability – understood as a probability, that user position error doesn’t exceed limit value defined arbitrary, at any moment in time $t$

3 states for railway track analyses:

- $U = 1$ cm – rail track deformation
- $U = 3$ cm – rail track inventory
- $U = 10$ cm – maximum acceptable position error
Typical realizations of the positioning process are characterized by the exponential distributions of the lifetime and the time of failures then:

**Positioning GNSS networking service availability:**

\[ A_{\text{exp}}(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} e^{-(\lambda+\mu)t} \]

... also we can calculate: Positioning GNSS networking service reliability and continuity

\[ R_{\text{exp}}(t, \tau) = \left[ \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} e^{-(\lambda+\mu)t} \right] \cdot e^{-\lambda \tau} \]

\[ \lim_{t \to \infty} R_{\text{exp}}(t, \tau) = \frac{\mu}{\lambda + \mu} \cdot e^{-\lambda \tau} \]

\[ C_{\text{exp}}(t, \tau) = e^{-\lambda \tau} \]

\[ \lim_{t \to \infty} C_{\text{exp}}(t, \tau) = e^{-\lambda \tau} \]

where \( \lambda, \mu \) are failure and renewal rates.
Availability functions and its limiting values

Positioning GNSS networking service availability function for 10 cm (brown)

Positioning GNSS networking service availability function for 3 cm (blue)

Positioning GNSS networking service availability function for 1 cm (red)
Trials in 2009-2010

RAILWAY ROUTE: KOŚCIERZYNA – KARTUZY
- 50 km, February 2009
GNSS Solution: GPS only, ASG-EUPOS Network, VRS, FKP, MAC.
Processing: Real Time, Service NAVGEO, Distance auto-recording 30 cm, Leica Office software.
Receivers: 4 phase GPS (Leica ATX 1230GG).

RAILWAY ROUTE: Gdańsk Central – Gdańsk Stogi
- 20 km, April 2010
GNSS Solution: GPS only, ASG-EUPOS Network, VRS, FKP, MAC.
Processing: Real Time, Service NAVGEO, Distance auto-recording 30 cm, Leica Office software.
Receivers: 3 phase GPS (Leica ATX 1230GG).

RAILWAY ROUTE: Gdańsk Osowa - Kościerzyna
60 km, November 2010
GNSS Solution: GPS only, ASG-EUPOS Network, VRS.
Processing: Real Time, Service NAVGEO, Distance auto-recording 30 cm, Leica Office software.
Receivers: 3 phase GPS (2x Leica ATX 1230GG, Leica GS 15).
First trials - 2009

1. 4 x SmartAntena ATX 1230GG with RX 1250Xc
2. Auto recording every 30 cm
3. Speed – 10 km/h
Availability results (first trials in 2009)

RAILWAY ROUTE: KOŚCIERZYNA – KARTUZY - 50 km,
February 2009 GNSS Solution: **GPS only**, ASG-EUPOS Network, VRS, FKP, MAC.
Processing: Real Time, Service NAVGEO, Distance auto-recording 30 cm, Leica Office software.
Receivers: 4 phase GPS (Leica ATX 1230GG).

Fixes with errors lower than 10 cm

4 receivers accuracy histograms

Availability functions for 10 cm position error limit
First trials (2009) – satellite availability

... to much errors...

- campaign planning (impossible for 5 hours of surveys)
- peoples on the platform
- receivers sky view
- terrain obstacles

How to minimize DOP’s?

\[
\min \{ trace(A) \} = \min \{ trace \left( (G^TG)^{-1} \right) \} = \min \left\{ \frac{1}{G^2} \sum_{i,j} (g_{i,j})^2 \right\}
\]
The surveyors abandoned the implementation of real-time measurements using the ASG-EUPOS, due to the existing gap in the work of the network associated with the transmission of GPS pseudorange corrections. In the afternoon, a large number of users caused a disconnection of service users packet data (GPRS).

The instability of the network ASG-EUPOS led the authors to the decision to resign from the measurements in real time. It was decided to implement them in post-processing, making development results gave greater freedom to use signals of different reference stations.

To improve the accuracy of the positions directly related to the number of available GPS satellites, it was decided to implement measurement using dual-mode GNSS receivers, thus utilizing the signals of two satellite systems: GPS and Glonass (new GNSS - private network was established in 2011).
**Trials in 2012-2014**

<table>
<thead>
<tr>
<th>TRAMP ROUTE: Gdańsk City, 50 km, <strong>February 2012</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>GNSS Solution: <strong>GPS/GLONASS</strong>, FKP from station located in Gdańsk.</td>
</tr>
<tr>
<td>Processing: Post-processing, Auto-recording every 30 cm, Leica Office software.</td>
</tr>
<tr>
<td>Receivers: 2 phase GPS/GLONASS (Leica GS 12, Leica GS 15).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TRAMP ROUTE: Gdańsk City 30 km, <strong>September 2013</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>GNSS Solution: <strong>GPS/GLONASS</strong>, VRS from Leica SmartNet GNSS network.</td>
</tr>
<tr>
<td>Processing: Real Time, Auto-recording 20 Hz (Leica GS 15), Leica Office software.</td>
</tr>
<tr>
<td>Receivers: 2 phase GPS/GLONASS (Leica GS 12, Leica GS 15).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TRAMP ROUTE: Narrow-Gauge Railway Koszalin 18 km, <strong>May 2014</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>GNSS Solution: <strong>GPS/GLONASS</strong>, VRS from Leica SmartNet GNSS network.</td>
</tr>
<tr>
<td>Processing: Real Time, Auto-recording 20 Hz, Leica Office software.</td>
</tr>
<tr>
<td>Receivers: 1 phase GPS/GLONASS (Leica GS 15).</td>
</tr>
<tr>
<td>Parameter for positioning</td>
</tr>
<tr>
<td>---------------------------</td>
</tr>
<tr>
<td>GNSS systems</td>
</tr>
<tr>
<td>Terrain type</td>
</tr>
<tr>
<td>Dimention</td>
</tr>
<tr>
<td>MTBF for $U &lt; 1$ cm [s]</td>
</tr>
<tr>
<td>Availability limiting value for $U &lt; 1$ cm [%]</td>
</tr>
<tr>
<td>Reliability limiting value for $U &lt; 1$ cm and $\tau = 300$ s [%]</td>
</tr>
<tr>
<td>Continuity limiting value for $U &lt; 1$ cm and $\tau = 300$ s [%]</td>
</tr>
<tr>
<td>MTBF for $U &lt; 3$ cm [s]</td>
</tr>
<tr>
<td>Availability limiting value for $U &lt; 3$ cm [%]</td>
</tr>
<tr>
<td>Reliability limiting value for $U &lt; 3$ cm and $\tau = 300$ s [%]</td>
</tr>
<tr>
<td>Continuity limiting value for $U &lt; 3$ cm and $\tau = 300$ s [%]</td>
</tr>
<tr>
<td>MTBF for $U &lt; 10$ cm [s]</td>
</tr>
<tr>
<td>Availability limiting value for $U &lt; 10$ cm [%]</td>
</tr>
<tr>
<td>Reliability limiting value for $U &lt; 10$ cm and $\tau = 300$ s [%]</td>
</tr>
<tr>
<td>Continuity limiting value for $U &lt; 10$ cm and $\tau = 300$ s [%]</td>
</tr>
</tbody>
</table>
How accurate? … straight and deformed railway route

- Points Accuracy with Active Geodetic Network – VRS solution (only GPS) approx. 1.5 cm accuracy
- It allows to define the places where the railay Route is deformed
- Trails with GPS/Glonass solution (made in March 2012) allowed to reduce position error to 0.8 - 1 cm, with higher availability (82% ambiguity solutions on 50 km tramp route in Gdańsk)

![Graph](image.png)
The existence of barriers restricting the use of the methodology applied for years - inability to increase the availability of assignments for the accuracy of the coordinate position of less than 1 cm.

Getting an availability of over 90% for the measurement error of 3 cm seems impossible, exceeding the availability of 50% of the assigned altitude for level 1 cm is also virtually impossible.

There is no doubt that on the basis of the above data the research team found itself having to seek alternative methods of achieving measurements that would allow increasing the availability of high-accuracy GNSS appointments referenced to levels 1 cm and 3 cm. Therefore, the research strategy has also been verified.

The new strategy was decided to rely on two elements:

- the use of the INS system to assist the positioning,
- the use, besides GPS and GLONASS systems, additional satellites of the BeiDou system in the process of determining the position.
Pomeranian Metropolitan Railway trials in 2015

TRAMP ROUTE: Pomeranian Metropolitan Railway, Gdańsk City, 60 km, June 2015


Project in the 12 year history of the Pomeranian Voivodeship, pointing out that never before in its history has the Pomeranian government undertaken a project costing over $200 million. The PKM has eleven (11) stations along the 19.5 km route. The line is not electrified, but is operated by ten diesel-powered trainsets which were ordered from Polish factory PESA SA of Bydgoszcz costing $34 million.
PKM 2015 measurement results

- three measures (availability, reliability and continuity) for levels 3 cm and 10 cm are **close to 100 %**
- For the accuracy level 1 cm the increase of accessibility in the horizontal plane is **very high (31.28 %)**
- The same is in the vertical plane (1D) - **31.34 %**.

<table>
<thead>
<tr>
<th>Parameter for positioning</th>
<th>PKM 2015 (no inertial system)</th>
<th>PKM 2015 (with inertial system)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GNSS systems</td>
<td>GPS / GLONASS</td>
<td>GPs / GLONASS / BeiDou</td>
</tr>
<tr>
<td>Terrain type</td>
<td>City approx. 500 000 citizens</td>
<td>City approx. 500 000 citizens</td>
</tr>
<tr>
<td>Dimenstions</td>
<td>3D</td>
<td>2D</td>
</tr>
<tr>
<td>MTBF for $U &lt; 1 \text{ cm} [s]$</td>
<td>2.06</td>
<td>35.44</td>
</tr>
<tr>
<td><strong>Availability limiting value for $U &lt; 1 \text{ cm} [%]</strong></td>
<td><strong>0.16</strong></td>
<td><strong>56.75</strong></td>
</tr>
<tr>
<td>Reliability limiting value for $U &lt; 1 \text{ cm}$ and $\tau = 300 \text{ s} [%]$</td>
<td>$3.762 \cdot 10^{-4}$</td>
<td>52.15</td>
</tr>
<tr>
<td>Continuity limiting value for $U &lt; 1 \text{ cm}$ and $\tau = 300 \text{ s} [%]$</td>
<td>23.36</td>
<td>91.88</td>
</tr>
<tr>
<td>MTBF for $U &lt; 3 \text{ cm} [s]$</td>
<td>134.90</td>
<td>$1.422 \cdot 10^3$</td>
</tr>
<tr>
<td><strong>Availability limiting value for $U &lt; 3 \text{ cm} [%]$</strong></td>
<td><strong>76.52</strong></td>
<td><strong>83.15</strong></td>
</tr>
<tr>
<td>Reliability limiting value for $U &lt; 3 \text{ cm}$ and $\tau = 300 \text{ s} [%]$</td>
<td>74.83</td>
<td>82.97</td>
</tr>
<tr>
<td>Continuity limiting value for $U &lt; 3 \text{ cm}$ and $\tau = 300 \text{ s} [%]$</td>
<td>97.80</td>
<td>99.78</td>
</tr>
<tr>
<td>MTBF for $U &lt; 10 \text{ cm} [s]$</td>
<td>$3.235 \cdot 10^3$</td>
<td>$4.859 \cdot 10^3$</td>
</tr>
<tr>
<td><strong>Availability limiting value for $U &lt; 10 \text{ cm} [%]$</strong></td>
<td><strong>84.62</strong></td>
<td><strong>85.39</strong></td>
</tr>
<tr>
<td>Reliability limiting value for $U &lt; 10 \text{ cm}$ and $\tau = 300 \text{ s} [%]$</td>
<td>84.54</td>
<td>85.34</td>
</tr>
<tr>
<td>Continuity limiting value for $U &lt; 10 \text{ cm}$ and $\tau = 300 \text{ s} [%]$</td>
<td>99.90</td>
<td>99.93</td>
</tr>
</tbody>
</table>
Summary 2009-2015

- Positioning GNSS networking service availability as a function of GNSS solution
- 6 years of measurement campaigns (2009-2015)
- and three levels of accuracies: 1 cm, 3 cm, 10 cm
Conclusions

- In the past 6 years the research team completed seven measurement campaigns aimed at the implementation of mobile measurement methods in the field of satellite geodesy for geodetic services of railway and tram lines.
- In the first years of the study (2009-2010) they sought to achieve such level of accuracy (3 cm) that would allow the possibility of implementing an inventory of railway lines for the project requirements. Initial tests were conducted based on GPS Network.
- From 2012 to 2014 they carried out measurements on the basis of GPS/GLONAS Network. It resulted in approx. **20-30% increase in the availability** of solutions with an error in horizontal and vertical plane not exceeding 3 cm.
- At the same time it was recognized that with careful planning to ensure a right constellation of satellites in time of taking measurements it is possible to achieve in a short period of time – the sub-centimeter accuracies, allowing to work on issues of diagnostics of track deformation. The availability of this solution was approx. 50% of the time of measurements.
- In 2015, the inventory research was decided to support GNSS measurements with the **inertial system**, and to use the GNSS network, which will provide a triple-system positioning based on GPS/GLONASS/BeiDou. As the research results demonstrated, the new strategy made it possible to **increase the availability of precision measuring range (1 cm and 3 cm)** by approx. **30 percent** in most dimensions of positioning.
THANK YOU